VISION OF THE INSTITUTE

- To be a university globally trusted for technical excellence where learning and research integrate to sustain society and industry.

MISSION OF THE INSTITUTE

- To offer undergraduate, postgraduate, doctoral and modular programmes in multi-disciplinary / inter-disciplinary and emerging areas.
- To create a converging learning environment to serve a dynamically evolving society.
- To promote innovation for sustainable solutions by forging global collaborations with academia and industry in cutting-edge research.
- To be an intellectual ecosystem where human capabilities can develop holistically.

VISION OF THE DEPARTMENT

- To be a global centre of excellence in mathematics and scientific computing for the growth of science and technology

MISSION OF THE DEPARTMENT

- Committed to the cause of quality education, research and consultancy by providing principled and highly skilled mathematics.

PROGRAMME EDUCATIONAL OBJECTIVES (PEOs)

PEO1	Graduates will achieve successful careers in academia, industry, or government sectors, demonstrating advanced knowledge and skills in mathematical sciences.
PEO2	Graduates will engage in lifelong learning and pursue advanced research and development in mathematics and related interdisciplinary fields.
PEO3	Graduates will assume leadership roles and contribute to solving complex problems in society through their expertise in mathematics and quantitative reasoning.

PROGRAMME OBJECTIVES (POs)

PO1	Students will acquire and demonstrate comprehensive understanding and proficiency in advanced mathematical theories, methods, and applications.
PO2	Students will develop strong analytical and problem-solving skills, enabling them to formulate and solve complex mathematical problems using appropriate techniques and tools.
PO3	Students will enhance their ability to communicate mathematical concepts effectively, both orally and in writing, and collaborate successfully in multidisciplinary teams to address real-world challenges.

CURRICULUM FRAMEWORK / FLEXIBLE CURRICULUM / NEP 2020 / M.Sc.

Components	Number of Courses	Number of Credits
Program Core (PC)	1st semester – 5 2nd semester – 4 3rd semester – 2	44
Program Elective (PE)	2nd semester – 1 3rd semester – 2 4th semester – 1	12
Essential Laboratory Requirements (ELR)	1st semester – 1 2nd semester – 1 3rd semester – 1	6
Seminar/Internship	1	2
Online course (OC) / Open Elective (OE) /Program Elective (PE)	4th semester – 2	6
Project Phase-I	3rd semester	2
Project Phase-II	4th semester	8
	Total Credits	**80**

M.Sc. Curriculum

Semester I

S.No.	Code	Course of Study	L	T	P	Credit
1.	MA701	Real Analysis	3	1	0	4
2.	MA702	Linear Algebra	3	1	0	4
3.	MA703	Ordinary Differential Equations	3	1	0	4
4.	MA704	Numerical Analysis	3	1	0	4
5.	MA705	Probability and Statistics	3	1	0	4
6.	MA706	R-Language	1	0	1	2
		Total Credits				**22**

Semester II

S.No.	Code	Course of study	L	T	P	Credit
1.	MA707	Abstract Algebra	3	1	0	4
2.	MA708	Complex Analysis	3	1	0	4
3.	MA709	Topology	3	1	0	4
4.	MA710	Partial Differential Equations	3	1	0	4
5.	MA 7**	Program Elective	3	0	0	3
6.	MA711	Scientific Computing using Python	1	0	1	2
		Total Credits				**21**

Semester III

S.No.	Code	Course of study	L	T	P	Credit
1.	MA712	Functional Analysis	3	1	0	4
2.	MA713	Operations Research	3	1	0	4
3.	MA7**	Program Elective	3	0	0	3
4.	MA7**	Program Elective	3	0	0	3
5.	MA714	Mathematical Software Lab	1	0	1	2
6.	MA715	Project Phase – I				2
7.	MA716	Seminar / Internship*				2
		Total Credits				**20**

Semester IV

S.No.	Code	Course of study	L	T	P	Credit
1.		Online Course# / Open Elective / Program Elective				6
2.	MA7**	Program Elective	3	0	0	3
3.	MA717	Project Phase – II				8
		Total Credits				**17**

TOTAL CREDITS: 80

* - Industry / Academic internship during the first year summer vacation for 6 – 8 weeks.
# - Online courses can be registered from first semester onwards.
PC – Programme Core
PE – Programme Elective
ELR – Essential Laboratory Requirement
OC – Online Course

Program Electives:

Sl. No.	Code	Course Name
Semester II		
1.	MA721	Integral Transforms
2.	MA722	Graph Theory
3.	MA723	Mathematics of Machine Learning (newly proposed)
Semester III		
1.	MA731	Fluid Dynamics
2.	MA732	Integral Equations and Calculus of Variations
3.	MA733	Measure Theory
4.	MA734	Optimization Techniques
5.	MA735	Stochastic Processes
Semester IV (6 options are kept since 2-3 PE will be offered in this semester)		
6.	MA741	Numerical Solution of Differential Equations
7.	MA742	Operator Theory
8.	MA743	Introduction of Fuzzy Mathematics and its applications
9.	MA744	Introduction to Singularly Perturbed Differential Equations
10.	MA745	Matrix Analysis
11.	MA746	Advanced Partial Differential Equations
Advanced Electives for PhD coursework		
12.	MA751	Nonlinear Programming
13.	MA752	Advanced Fuzzy Mathematics and its Applications
14.	MA753	Matrix Theory and Stochastic Programming

15.	MA754	Advanced Numerical Analysis for Singularly Perturbed Differential Equations
16.	MA755	Bio-rheology
17.	MA756	Advanced Complex Analysis
18.	MA757	Geometric Function Theory
19.	MA758	Approximation Theory
20.	MA759	Convex Analysis
21.	MA760	Graphs and Matrices
22.	MA761	Fitted Mesh and Fitted Operator Methods for Singular Perturbation problems
23.	MA762	Queueing Theory
24.	MA763	Finite Element Methods
25.	MA764	Particulate Processes: Theory and Modelling
26.	MA765	Introduction to Hydrodynamics Stability
27.	MA766	Fixed Point Theory and its Applications
28.	MA767	Advanced Functional Analysis
29.	MA768	Theory and Geometry of Banach Spaces
30.	MA769	Applied Functional Analysis
31.	MA781	Research Methodology in Mathematical Sciences
32.	MA782	Advanced Mathematics

COURSE OUTCOME AND PROGRAMME OUTCOME MAPPING

PROGRAMME CORE

Course Codes	Course Title	CO	Course Outcomes: At the end of the course student will be able to	PO1	PO2	PO3
MA 701	Real Analysis	CO1	analyze convergent sequences, Cauchy sequences, continuous functions and uniform continuous functions in metric spaces.	3	3	2
		CO2	verify whether a metric space is complete, connected or compact and apply the properties of these metric spaces to solve theoretical problems.	3	3	2
		CO3	determine convergence of sequence/series of functions by using various tests and the sum of a convergent series of functions.	3	3	2
		CO4	state, prove and apply key theorems in the space of continuous functions in different cores like approximation theory and functional analysis.	3	3	2
		CO5	understand and evaluate the Lebesgue measure and integration on the real line.	3	3	2
MA 702	Linear Algebra	CO1	identify bases for vector spaces, execute change of bases and	3	3	2

			understand their effects on matrix representations.			
		CO2	compute eigenvalues and eigenvectors for linear transformations, perform diagonalization on matrices and apply Cayley-Hamilton theorem to find the minimal polynomial of a matrix.	3	3	2
		CO3	utilize invariant direct sums in matrix analysis and transform linear transformations and matrices into Jordan canonical forms.	3	3	2
		CO4	determine a linear operator is normal, unitary, and orthogonal projection and to construct the spectral decomposition of normal and self-adjoint operators.	3	3	2
		CO5	construct the matrix of a bilinear form, study the properties of bilinear forms and apply Sylvester's law of inertia to quadratic forms.	3	3	2
MA 703	Ordinary Differential Equations	CO1	state and interpret the existence and uniqueness theorems.	3	3	2
		CO2	calculate the general solution to a homogeneous linear differential equation with constant coefficients.	3	3	2
		CO3	apply series solution techniques to differential equations.	3	3	2
		CO4	utilize the method of variation of parameters to solve linear non-homogeneous differential equations with variable coefficients	3	3	2
		CO5	perform stability analysis of the equilibrium point in an autonomous system.	3	3	2
MA 704	Numerical Analysis	CO1	find the roots of transcendental and polynomial equations.	3	3	2
		CO2	compute the numerical solution of system of linear equations and find dominant eigenvalues of the matrix.	3	3	2
		CO3	use various interpolation methods for the data.	3	3	2
		CO4	find numerical differentiation and numerical integration.	3	3	2
		CO5	compute approximate solution of ordinary and partial differential equations.	3	3	2
MA 705		CO1	understand the axiomatic rudiments of modern probability theory and use of	3	3	2

	Probability and Statistics		random variables as an intrinsic tool for the analysis of random phenomena.			
		CO2	characterize multiple input and output system probability models based on single & multiple random variables.	3	3	2
		CO3	evaluate moments, characteristic functions and understand the concept of inequalities and probabilistic limits	3	3	2
		CO4	compute point estimators and interval estimation for various statistical measures.	3	3	2
		CO5	use basic statistical knowledge in testing of hypotheses on large and small samples.	3	3	2
MA 707	Abstract Algebra	CO1	demonstrate a solid understanding of advanced group theory concepts.	3	3	2
		CO2	apply the fundamental theorem of finite abelian groups to solve problems involving group structure.	3	3	2
		CO3	understand and utilize the ideals, factor rings, work within Euclidean domains, principal ideal domains, and unique factorization domains.	3	3	2
		CO4	develop a comprehensive understanding of field extensions, including splitting fields, algebraic and transcendental extensions, simple extensions, separable extensions, and finite fields.	3	3	2
		CO5	comprehend the fundamental theorem of Galois theory and its implications for the correspondence between field extensions and group theory.	3	3	2
MA 708	Complex Analysis	CO1	analyze properties of analytic functions and construct branch of logarithm.	3	3	2
		CO2	determine properties of Mobius transformation and express analytic functions as power series.	3	3	2
		CO3	prove various forms of Cauchy's integral formula/theorems.	3	3	2
		CO4	find singularities and residues at them as well as obtain Laurent's series expansions.	3	3	2
		CO5	evaluate real integrals using Cauchy's residue theorem and analyze location of zeros of analytic functions.	3	3	2
MA 709	Topology	CO1	understand various basic concepts like open sets, basis for a topology, closed	3	3	2

			sets, product topology, order topology and solve related problems.			
		CO2	understand continuous functions and construction of continuous functions, homeomorphism, metric topology, quotient topology.	3	3	2
		CO3	understand connectedness and components and theorems about connectedness, local connectedness and path connectedness and do related problems.	3	3	2
		CO4	understand compactness, limit point compactness, local compactness, compactifications and countability axioms and prove their properties.	3	3	2
		CO5	understand all separation axioms, Hausdorff spaces, Regular, Normal spaces and prove theorems on separation axioms, Urysohn's lemma, Urysohn's metrization theorem, Tietze extension theorem and Tychonoff theorem.	3	3	2
MA 710	Partial Differential Equations	CO1	solve first-order linear and nonlinear partial differential equations using the method of characteristics.	3	3	2
		CO2	recognize and reduce the second-order partial differential equations to its canonical form.	3	3	2
		CO3	model the classical wave equations of mathematical physics and discuss their solutions	3	3	2
		CO4	identify the classical diffusion equations and discuss their solutions.	3	3	2
		CO5	solve elliptic equations using method of separation of variables.	3	3	2
MA 712	Functional Analysis	CO1	understand various concepts of normed linear spaces and construct new spaces from old ones	3	3	2
		CO2	find duals of standard spaces and prove fundamental results about bounded linear functionals.	3	3	2
		CO3	develop a comprehensive understanding of the major theorems of functional analysis and their applications.	3	3	2
		CO4	evaluate orthogonal decomposition of Hilbert spaces and prove several other results.	3	3	2

		CO5	comprehend projection theorem and Riesz representation theorem.			
MA 713	Operations Research	CO1	solve linear programming problem using simplex, Big-M and two-phase methods.	3	3	2
		CO2	perform sensitivity analysis and apply duality theory.	3	3	2
		CO3	identify and evaluate transportation and assignment problems.	3	3	2
		CO4	utilize the solutions techniques for dynamic problems.	3	3	2
		CO5	analyze and solve nonlinear programming problems	3	3	2

LABORATORY

Course Codes	Course Title	CO	Course Outcomes: At the end of the course student will be able to	PO1	PO2	PO3
MA 706	R Language	CO1	explain various data types and data structures in R.	2	3	2
		CO2	write functions using control statements and loops.	2	3	2
		CO3	compute various statistical measures using R.	2	3	2
		CO4	generate various plots for the given data.	2	3	2
		CO5	perform various manipulations with date and time.	2	3	2
MA 711	Scientific Computing using Python	CO1	develop basic proficiency in Python programming.	2	3	2
		CO2	understand and use various Python modules.	2	3	2
		CO3	perform basic mathematical computations using Python.	2	3	2
		CO4	create basic plots and visualizations.	2	3	2
		CO5	develop good coding practices and use Python to solve introductory-level mathematical problems.	2	3	2
MA 714	Mathematical Software Lab	CO1	perform basic operations and symbolic calculations in SCILAB.	2	3	2
		CO2	understand and use in-built functions.	2	3	2
		CO3	create SCILAB codes to solve problem in ordinary differential equations, linear algebra, trigonometry.	2	3	2
		CO4	create basic shapes using TikZ and typeset mathematical theorems and proofs using LaTeX.	2	3	2

		CO5	format research articles, books, theses and create slides for presentations using the beamer class.	2	3	2

PROGRAM ELECTIVES (PE)

Semester II

Course Codes	Course Title	CO	Course Outcomes: At the end of the course student will be able to	PO1	PO2	PO3
MA721	Integral Transforms	CO1	compute Laplace transforms of various functions and their inverses.	3	3	2
		CO2	use the Fourier series to decompose periodic functions and solve related problems.	3	3	2
		CO3	find Fourier transforms of elementary functions.	3	3	2
		CO4	apply transform techniques to solve ordinary and partial differential equations.	3	3	2
		CO5	employ Z-transforms for solving difference equations.	3	3	2
MA722	Graph Theory	CO1	find minimum spanning trees and shortest paths in graphs.	3	3	2
		CO2	apply Tutte’s 1-factor theorem to solve problems related to bipartite matchings.	3	3	2
		CO3	solve connectivity problems in graphs and digraphs.	3	3	2
		CO4	explore Turan’s theorem to solve graph coloring problems and characterize planar graphs.	3	3	2
		CO5	evaluate Eulerian trails and apply approximate solutions for the travelling salesman problem in practical scenarios.	3	3	2
MA723	Mathematics of Machine Learning	CO1	use probability concepts and Bayesian inference for machine learning	2	3	2
		CO2	implement gradient descent and convex optimization to train machine learning models.	2	3	2
		CO3	create and assess linear and logistic regression models with regularization for prediction and classification.	2	3	2
		CO4	manage bias-variance tradeoff and use model selection and cross-validation to improve model performance.	2	3	2

		CO5	build and optimize deep learning models, including CNNs and RNNs, for tasks such as image recognition and natural language processing.	2	3	2

Semester III

Course Codes	Course Title	CO	Course Outcomes: At the end of the course student will be able to	PO1	PO2	PO3
MA731	Fluid Dynamics	CO1	understand physical concepts involved in fluid motion.	3	3	2
		CO2	model some two dimensional inviscid flow problems through source/sink doublet.	3	3	2
		CO3	find solution of some three dimensional inviscid flows.	3	3	2
		CO4	identify the physics involved in a given viscous flow and simplify Navier-Stoke's equation to find solution.	3	3	2
		CO5	find exact solution of some fluid flow problems and interpret results physically.	3	3	2
MA732	Integral Equations and Calculus of Variations	CO1	explain basic concepts of Volterra and Fredholm integral equations and their relationship with ordinary differential equations.	3	3	2
		CO2	apply the method of successive approximations and resolvent kernels to solve integral equations.	3	3	2
		CO3	construct Green's functions for boundary value problems.	3	3	2
		CO4	apply Euler-Lagrange equations to solve natural boundary and transversality conditions problems, and analyze sufficient conditions for extremum.	3	3	2
		CO5	design and evaluate variational methods for time-dependent problems using approximation techniques such as Galerkin and weighted-residual methods.	3	3	2

MA733	Measure Theory	CO1	apply the concepts of rings, algebras, sigma-algebras, and Borel sets in the context of measure theory and apply Egorov's and Lusin's theorems in the analysis of measurable functions.	3	3	2
		CO2	compute the Lebesgue integral for non-negative measurable and integrable functions and utilize key theorems of measure theory in problem solving.	3	3	2
		CO3	compare and contrast Riemann and Lebesgue integration, understanding the criteria for Lebesgue integrability of Riemann integrable functions and the treatment of improper Riemann integrals.	3	3	2
		CO4	analyse functions of bounded variation, understand their indefinite integrals and apply Vitali's lemma. Also, demonstrate knowledge of the properties and characterizations of absolutely continuous functions, including their differentiation and integration.	3	3	2
		CO5	verify the completeness of standard measurable spaces, and apply concepts related to signed measures, including the Hahn and Jordan decompositions and the Radon-Nikodym theorem.	3	3	2
MA734	Optimization Techniques	CO1	analyze convex sets and convex functions and to compute subgradients of a convex function.	3	3	2
		CO2	verify optimality conditions of an unconstrained optimization problem and to solve it using steepest descent, Newton’s and conjugate gradient methods.	3	3	2
		CO3	find an optimum solution to a constrained optimization problem using Karush-Kuhn-Tucker and Fritz John conditions.	3	3	2

		CO4	determine an optimal solution of constrained optimization problem using Frank-Wolfe's, projected gradient and penalty methods.	3	3	2
		CO5	write Lagrangian dual problem to a non-linear programming and apply duality results to solve it.	3	3	2
MA735	Stochastic Processes	CO1	understand the basic concepts and classifications of stochastic processes.	3	3	2
		CO2	explain the various renewal processes.	3	3	2
		CO3	describe Markov chains and classification of states.	3	3	2
		CO4	analyze discrete-time Markov chain.	3	3	2
		CO5	compute the mean first passage time for a CTMC.	3	3	2

Semester IV

Course Codes	Course Title	CO	Course Outcomes: At the end of the course student will be able to	PO1	PO2	PO3
MA741	Numerical Solution of Differential Equations	CO1	apply finite difference method to solve boundary value problem in ordinary differential equations.	3	3	2
		CO2	design finite difference numerical scheme for parabolic equations and solve numerically.	3	3	2
		CO3	solve hyperbolic equations through method of characteristics and Lax-Wendroff method.	3	3	2
		CO4	find numerical approximations to the solution of Laplace and Poisson equations in Cartesian and polar coordinates.	3	3	2
		CO5	construct integral formulations for the given boundary value problem and solve them using FEM.	3	3	2
MA742	Operator Theory	CO1	represent and analyse the duals of various normed spaces and use the concepts of reflexivity and weak/weak* convergences.	3	3	2

		CO2	apply the properties of adjoint, self-adjoint, normal, and unitary operators in Hilbert spaces and use these properties in problem-solving.	3	3	2
		CO3	describe and utilize projection operators in Banach and Hilbert spaces, and understand their significance in various applications.	3	3	2
		CO4	solve problems involving eigenvalues, eigenvectors, eigenspaces, and invariant spaces and apply the spectral theorem in finite-dimensional Hilbert spaces.	3	3	2
		CO5	apply spectral theory to Banach and Hilbert space operators, including the analysis of the eigen spectrum, approximate eigen spectrum, spectrum and resolvent, and the spectral radius formula.	3	3	2
MA743	Introduction to Fuzzy Mathematics and its Applications	CO1	analyze different types of fuzzy sets, α-cuts and its properties and extension of functions.	3	3	2
		CO2	apply the operations (fuzzy complements, fuzzy intersections, and fuzzy unions) on fuzzy sets.	3	3	2
		CO3	create the fuzzy relations and identify the different types of fuzzy relations and their applications.	3	3	2
		CO4	implement fuzzy relations, fuzzy c-means methods to clustering of data (information) in engineering problems.	3	3	2
		CO5	use the concepts of fuzzy decision-making methods in engineering and management problems.	3	3	2
MA744	Introduction to Singularly Perturbed	CO1	determine breakdown points for asymptotic expansions.	3	3	2
		CO2	derive the asymptotic	3	3	2

	Differential Equations		approximations for solutions of singular perturbation problems.			
		CO3	deduce the boundary layer behavior of solution of singular perturbation problems.	3	3	2
		CO4	use matched asymptotic method to solve two -point boundary value problems.	3	3	2
		CO5	apply the WKB method to solve singular perturbation problems.	3	3	2
MA745	Matrix Analysis	CO1	determine the determinant and inverse of a partitioned matrix.	3	3	2
		CO2	construct Jordan canonical form and singular value decomposition of a matrix.	3	3	2
		CO3	derive matrix inequalities using Schur complements, Hadamard and Kronecker products.	3	3	2
		CO4	identify positive semidefinite matrices through eigenvalues, minors, and determinants and be able to compute Perron vector.	3	3	2
		CO5	categorize matrix classes and use them in stability analysis and complementarity problems.	3	3	2
MA746	Advanced Partial Differential Equations	CO1	understand the basic concepts and classifications of partial differential equations.	3	3	2
		CO2	illustrate the conservation laws and different solution methodologies.	3	3	2
		CO3	explain Sobolov spaces and its various properties.	3	3	2
		CO4	analyze weak solutions for different types problems.	3	3	2
		CO5	use non-variational techniques for the solution methodologies.	3	3	2

Advanced Electives

Course Codes	Course Title	CO	Course Outcomes: At the end of the course student will be able to	PO1	PO2	PO3
MA751	Non-linear Programming	CO1	determine convex sets and convex functions and to solve convex programming problems.	3	3	2
		CO2	verify the optimality conditions and to apply steepest descent, Newton's and conjugate gradient methods to solve unconstrained optimization problems.	3	3	2
		CO3	use the method of Lagrange multipliers and Karush-Kuhn-Tucker conditions to find an optimum solution of a nonlinear programming problems.	3	3	2
		CO4	solve constrained nonlinear programming using Frank-Wolf's, projected gradient, Beale and penalty methods.	3	3	2
		CO5	construct the dual of a non-linear programming problem and to apply duality results to solve them.	3	3	2
MA752	Advanced Fuzzy Mathematics and its Applications	CO1	apply operations on fuzzy numbers such as addition, subtraction, multiplication, division, fuzzy max and fuzzy min.	3	3	2
		CO2	identify different methods on ranking of fuzzy numbers using degree of optimality	3	3	2
		CO3	compare different methods on ranking of fuzzy numbers using comparison function and scoring functions.	3	3	2
		CO4	illustrate different fuzzy decision-making methods using arithmetic operations	3	3	2
		CO5	classify different fuzzy decision-making methods using maxmin	3	3	2

			composition and converting fuzzy numbers to crisp scores.			
MA753	Matrix Theory and Stochastic Programming	CO1	understand probability theory in depth.	3	3	2
		CO2	explain the random vectors and distributions.	3	3	2
		CO3	utilize matrix theory for linear programming.	3	3	2
		CO4	work with vector spaces and inner product spaces.	3	3	2
		CO5	solve the discrete moment problems using linear programming.	3	3	2
MA754	Advanced Numerical Analysis for Singularly Perturbed Differential Equations	CO1	design numerical methods with fitting factors for singularly perturbed initial value problems.	3	3	2
		CO2	construct and analyse fitting factors for singularly perturbed boundary value problems.	3	3	2
		CO3	solve various classes of singular perturbation problems numerically.	3	3	2
		CO4	prove convergence of fitted mesh methods for one and two dimensional singular perturbation problems.	3	3	2
		CO5	use finite element and finite volume methods for solving singular perturbation problems.	3	3	2
MA755	Bio-rheology	CO1	identify the types of fluids and flows.	3	3	2
		CO2	figure out the blood components and the factors affecting blood.	3	3	2
		CO3	choose research problem based on the size of blood vessel.	3	3	2
		CO4	apply ideas of oscillatory flow and pulsatile flow in their research problems.	3	3	2
		CO5	select research problems on permeability.	3	3	2

MA756	Advanced Complex Analysis	CO1	prove properties of convex functions and prove Hadamard/Phragmen Lindelof theorems.	3	3	2
		CO2	explore various theorems about normal families and use them to prove Riemann mapping theorem.	3	3	2
		CO3	obtain infinite products and characterise simply connected domains	3	3	2
		CO4	identify various properties of harmonic functions.	3	3	2
		CO5	analyze various properties of entire functions and Picard theorems.	3	3	2
MA757	Geometric Function Theory	CO1	prove basic properties of univalent functions including those with special geometric properties.	3	3	2
		CO2	derive Lower’s differential equations and to use them to investigate univalent functions.	3	3	2
		CO3	analyze Grunsky, Goluzin, Lebedev inequalities and other related inequalities.	3	3	2
		CO4	define subordination and majorization and use them to prove coefficient inequalities.	3	3	2
		CO5	use the theory of differential subordination to prove properties of univalent functions.	3	3	2
MA758	Approximation Theory	CO1	explain the approximation of continuous functions by polynomials.	3	3	2
		CO2	find some applications of Chebyshev alternation theorem.	3	3	2
		CO3	comprehend the best approximation from closed convex subsets of inner product spaces andcontinuity of metric projection.	3	3	2
		CO4	understand the best approximation by subspaces of Banach spaces.	3	3	2

		CO5	explain the semi continuity properties of set-valued maps and continuous selection of metric projection maps.	3	3	2
MA759	Convex Analysis	CO1	explain the convex sets, Caratheodory's theorem and Helley's theorem.	3	3	2
		CO2	understand the directional derivatives and sub gradients of convex functions.	3	3	2
		CO3	comprehend the extreme points of convex sets and know some applications of Krein-Milman theorem.	3	3	2
		CO4	find Gateaux and Fréchet derivatives of convex functions defined on Banach spaces.	3	3	2
		CO5	explain the derivatives of norm functions and duality maps.	3	3	2
MA760	Graphs and Matrices	CO1	compute determinant and inverse of a block matrix using Schur complement	3	3	2
		CO2	apply Perron-Frobenius theorem and the Rayleigh quotient to several graph matrices to obtain bounds for graph theoretic parameters	3	3	2
		CO3	give the rank and the column space of the Incidence matrix of a graph and derive eigenvalue inequalities between matrices of graphs and its induced subgraphs using interlacing theorem	3	3	2
		CO4	find coefficients of the characteristic polynomial of the adjacency matrix and relate cofactors of the Laplacian matrix of a graph with its spanning trees	3	3	2
		CO5	determine inertia, determinant, and inverse formula for the distance matrix of a tree.	3	3	2
MA761	Fitted Mesh and Fitted Operator Methods for Singular	CO1	determine solution behaviour of singular perturbation problems.	3	3	2
		CO2	design fitted mesh methods for reaction-diffusion equations.	3	3	2
		CO3	construct fitted mesh methods for convection-diffusion equations.	3	3	2

	Perturbation Problems	CO4	derive error estimates of numerical schemes with constant fitting factors.	3	3	2
		CO5	solve boundary value problems with constant fitting factors.	3	3	2
MA762	Queueing Theory	CO1	analyze the birth and death processes and their applications.	3	3	2
		CO2	derive various performance measures of the queueing model with general service times.	3	3	2
		CO3	explain continuous phase-type distribution and BMAP.	3	3	2
		CO4	illustrate matrix analytic method for various queueing models	3	3	2
		CO5	construct queueing systems from real-life.	3	3	2
MA763	Finite Element Methods	CO1	demonstrate the ability to solve elliptic problems using the Finite Element Method.	3	3	2
		CO2	analyze and estimate errors in FEM solutions.	3	3	2
		CO3	apply semi-discretization and fully discretization methods effectively to solve parabolic problems.	3	3	2
		CO4	implement and use stabilization techniques for accurately solving hyperbolic problems with FEM.	3	3	2
		CO5	implement DG methods for one-dimensional model problems and evaluate their convergence.	3	3	2
MA764	Particulate Processes: Theory and Modelling	CO1	understand and model different particulate events arising in industry.	3	3	2
		CO2	explore various population balance models and their relation with real-life.	3	3	2
		CO3	analyze different aspects of solutions (existence, uniqueness, steady-state, self-similarity, non-existence).	3	3	2

		CO4	design efficient and accurate numerical methods for solving coupled problems.	3	3	2
		CO5	prove stability of the numerical scheme and convergence of the solutions	3	3	2
MA765	Introduction to Hydrodynamic Stability	CO1	develop an understanding by which "small" perturbations grow, saturate and modify fluid flows.	3	3	2
		CO2	apply linear stability to determine the onset of instability.	3	3	2
		CO3	determine the nature of instabilities.	3	3	2
		CO4	recognize importance of unstable nonlinear solutions.	3	3	2
		CO5	build a basic foundation for addressing/approaching research problems on laminar to turbulent transition.	3	3	2
MA766	Fixed Point Theory and its Applications	CO1	explain the Banach contraction principle and its extensions. Apply the principle to solve systems of linear equations, differential equations, and integral equations.	3	3	2
		CO2	describe Caristi's fixed point theorem and the Ekeland principle. Analyze and find fixed points of non-expansive and set-valued maps. Understand and apply Brouwer-Schauder fixed point theorems	3	3	2
		CO3	explain Ky Fan's best approximation theorems in Hilbert spaces. Apply these theorems to fixed point problems and understand Prolla's theorem and its extensions.	3	3	2
		CO4	apply the KKM-Map principle and its extensions. Use the variants of the KKM-Map principle to solve practical problems and explore their	3	3	2

			applications.			
		CO5	analyze fixed point theorems in partially ordered spaces and other abstract spaces. Apply fixed point theory to problems in game theory and other relevant fields.	3	3	2
MA767	Advanced Functional Analysis	CO1	describe locally convex spaces and identify convex sets and semi-norms. Explain projective and inductive topologies and the separation theorems.	3	3	2
		CO2	apply the Hahn-Banach separation theorem, Banach-Alaoglu theorem, Goldstine's theorem, and Krein-Milman's theorem to solve problems in normed linear spaces.	3	3	2
		CO3	utilize Mazur's basic sequence theorem and block basis sequences in problem-solving. Investigate subspaces, complemented subspaces and special properties of various sequence spaces.	3	3	2
		CO4	explain and analyze linear operators on Banach spaces, including their adjoints, compact and finite rank operators, Fredholm operators, and weakly compact operators.	3	3	2
		CO5	describe and apply the principles of spectral theory for operators on Banach spaces. Analyze the structure of ideals and quotients in this context.	3	3	2
MA768	Theory and Geometry of Banach Spaces	CO1	apply the Banach-Alaoglu theorem and Goldstine's theorem to solve problems involving dual spaces and weak topologies.	3	3	2
		CO2	apply Krein-Milman's theorem, Eberlein-Smulian theorem, and Bishop-Phelps theorem to problems	3	3	2

			involving extreme points and convex sets.			
		CO3	explain and apply the concepts of rotundity, modulus of rotundity, and uniform rotundity. Use Milman-Pettis theorem to solve related problems.	3	3	2
		CO4	understand and apply smoothness concepts, including modulus of smoothness and spherical image maps.	3	3	2
		CO5	apply knowledge of ball intersection properties, M-ideals, M-summands, and L-summands to solve relevant problems and discuss their theoretical implications.	3	3	2
MA769	Applied Functional Analysis	CO1	identify and analyze test functions and related distributions.	3	3	2
		CO2	analyze various functions and theorems.	3	3	2
		CO3	explain different function spaces.	3	3	2
		CO4	analyze the solutions of boundary value problems in one dimension.	3	3	2
		CO5	use the solution strategies to solve problems in higher dimensions.	3	3	2
MA781	Research Methodology in Mathematical Sciences	CO1	understand research ethics, plagiarism and intellectual property rights.	3	3	3
		CO2	analyse multi-variable functions, metric spaces, holomorphic functions, groups, rings and fields.	3	3	3
		CO3	solve multiple integrals and higher order/simultaneous differential equations.	3	3	3
		CO4	simulate mathematical problems using software.	3	3	3
		CO5	prepare manuscripts and slides using LaTeX.	3	3	3
MA782	Advanced Mathematics	CO1	solve problems involving functions of several variables and related theorems.	3	3	2
		CO2	apply key theorems of metric spaces and complex analysis to solve mathematical problems.	3	3	2
		CO3	utilize and understand algebraic structures in group, ring and field	3	3	2

			theory.			
		CO4	solve differential equations numerically with error analysis and stability.	3	3	2
		CO5	apply principles of probability and statistics to random variables, and testing of hypothesis.	3	3	2

3 – High; 2 – Medium; 1 – Low

Course Code	MA701
Title of the Course and Type	Real Analysis (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. introduce the concept of metric spaces and to investigate convergent sequences and continuous functions in metric spaces.
2. explore the topics of connectedness, completeness and compactness thoroughly.
3. study the pointwise and uniform convergence of sequences and series of functions.
4. discuss the space of continuous functions and key results in approximation theory and equicontinuity.
5. introduce the Lebesgue measure and integration on the real line.

Course Content:

Review of properties of real numbers – Metric spaces – Basic concepts – Cauchy and convergent sequences in metric spaces – Continuity and uniformly continuity in metric spaces – Homeomorphisms – Complete metric spaces – The nested set theorem – Baire category theorem – Banach's contraction principle.

Connectedness – Intermediate value theorem – Continuity and connectedness – Separable metric spaces – Totally bounded sets – Compactness – Continuous functions on compact metric spaces – Heine-Borel theorem.

Sequences and series of real valued functions – Pointwise and uniform convergence – Cauchy's criterion – Tests for uniform convergence of sequence/series of functions – Weierstrass's M test – Abel's test – Dirichlet's test – Uniform convergence and continuity (Dini's theorem) – Uniform convergence and integration/differentiation.

Space of continuous functions – Bernstein polynomials – Weierstrass approximation theorem – Equicontinuity – Uniform boundedness – Arzela-Ascoli theorem – Completion of a metric space.

Lebesgue outer measure – Measurable sets – Non-measurable sets – Measurable functions – Lebesgue integration on the real line – Lebesgue's dominated convergence theorem – Fatou's lemma – Monotone convergence theorem.

Reference Books:

1. N. L. Carothers, *Real Analysis*, 1st edition, Cambridge University Press, 2009.
2. H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, 4th edition, Pearson Education, 2015.
3. W. Rudin, *Principles of Mathematical Analysis*, 3rd edition, Mc-Graw Hill, 2023.
4. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Indian edition, Mc-Graw Hill, 2017.
5. R. R. Goldberg, *Methods of Real Analysis*, reprint edition, Oxford and IBH, 2019.

Course Outcomes: On completion of the course, students will be able to

1. analyze convergent sequences, Cauchy sequences, continuous functions and uniform continuous functions in metric spaces.
2. verify whether a metric space is complete, connected or compact and apply the properties of these metric spaces to solve theoretical problems.
3. determine convergence of sequence/series of functions by using various tests and the sum of a convergent series of functions.
4. state, prove and apply key theorems in the space of continuous functions in different cores like approximation theory and functional analysis.
5. understand and evaluate the Lebesgue measure and integration on the real line

Course Code	MA702
Title of the Course and Type	Linear Algebra (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. teach the basic concepts of vector spaces and system of linear equations in detail.
2. study diagonalizable operator on a vector space and characterizations of it using the minimal and characteristic polynomials.
3. discuss various decompositions of vector spaces and linear transformations on vector spaces.
4. introduce different classes of linear operators on inner product spaces, their structures.
5. explore the concepts of bilinear and quadratic forms on vector spaces and Sylvester's law of inertia.

Course Content:

Review of basic concepts: Vector spaces, bases, dimension – Linear transformations – Matrix representation – Change of basis – Rank-nullity theorem – System of linear equations.

Characteristic values and characteristic vectors of linear transformations – Eigenspaces – Algebraic and geometric multiplicities of eigenvalues – Diagonalization – Minimal polynomial – Cayley-Hamilton theorem.

Invariant subspaces – Triangular form – Direct sum decompositions – Invariant direct sums – Primary decomposition theorem – Jordan canonical form.

Basic review of inner product spaces – Adjoint operators – Normal operators – Unitary operators – Orthogonal projections – Diagonalization of normal matrices – Spectral decompositions and spectral theorem – Applications.

Bilinear forms – Matrix representation – Quadratic forms – Positive definite forms – Sylvester's law of Inertia.

Reference Books:

1. K. Hoffman and R. Kunze. *Linear Algebra*, 2nd edition, Pearson, 2015.
2. S. H. Friedberg, A. J. Insel and L. E. Spence, *Linear Algebra*, 5th edition, Pearson, 2022.
3. S. Axler, *Linear Algebra Done Right*, 3rd edition, Springer, 2014.
4. S. Roman, A*dvanced Linear Algebra,* 3rd edition, Springer, 2008.
5. S. Kumaresan, *Linear Algebra: A Geometric Approach*, 1st edition, PHI Learning, 2001.

Course Outcomes: On completion of the course, students will be able to

1. identify bases for vector spaces, execute change of bases and understand their effects on matrix representations.
2. compute eigenvalues and eigenvectors for linear transformations, perform diagonalization on matrices and apply Cayley-Hamilton theorem to find the minimal polynomial of a matrix.
3. utilize invariant direct sums in matrix analysis and transform linear transformations and matrices into Jordan canonical forms.
4. determine a linear operator is normal, unitary, and orthogonal projection and to construct the spectral decomposition of normal and self-adjoint operators.
5. construct the matrix of a bilinear form, study the properties of bilinear forms and apply Sylvester's law of inertia to quadratic forms.

Course Code	MA703
Title of the Course and Type	Ordinary Differential Equations (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: Objectives of this course are to

1. introduce existence and uniqueness theorems and their uses.
2. discuss various approaches to finding general solutions to ordinary differential equations.
3. teach properties of special functions in the discussion of the solution of ordinary differential equations.
4. explain the criteria for oscillation and non-oscillation of solutions of certain linear differential equations in real domain.
5. familiarize with equilibrium solutions of autonomous first-order differential equations and determine their stability.

Course Content:

Review on the fundamentals of ordinary differential equations – Existence-uniqueness theorems (Picard's theorem, Peano's theorem) – Gronwall's inequality – Continuation of solutions and maximal interval of existence.

Euler-Cauchy equations – Solution by undetermined coefficients – Solution by variation of parameters – Basic concepts and theory of system of ordinary differential equations – Homogeneous and nonhomogeneous systems with constant coefficients.

Series solutions of first and second order linear equations – Ordinary and regular singular points – Gauss's hypergeometric equation – Point at infinity – Legendre polynomials – Bessel functions – Properties of Legendre Polynomials and Bessel functions.

Oscillation theory and boundary value problems – Qualitative properties of solutions – Sturm comparison theorems – Eigenvalues, eigen functions and the vibrating string.

Nonlinear equations – Autonomous systems – Phase plane and its phenomena – Types of critical points – Stability – Critical points and stability for linear systems – Stability by Lyapunov's direct method – Simple critical points of nonlinear systems.

Reference Books:

1. E. A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover publication, 2012.
2. M. Hirsch, S. Smale and R. Deveney, *Differential Equations, Dynamical Systems and Introduction to Chaos*, Academic Press, 2004.
3. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGraw Hill, 2016.
4. M. E. Taylor, *Introduction to Differential Equations*, American Mathematical Society (Indian edition), 2011.
5. W. E. Boyce, R. C. Di Prima and D. B. Meade, *Elementary Differential Equations and Boundary Value Problems*, Wiley, 2017.

Course Outcomes: On completion of the course, students will be able to

1. state and interpret the existence and uniqueness theorems.
2. calculate the general solution to a homogeneous linear differential equation with constant coefficients.
3. apply series solution techniques to differential equations.
4. utilize the method of variation of parameters to solve linear non-homogeneous differential equations with variable coefficients.
5. perform stability analysis of the equilibrium point in an autonomous system.

Course Code	MA704
Title of the Course and Type	Numerical Analysis (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. introduce various root-finding methods and discuss their convergence.
2. discuss direct and indirect methods for solving system of linear equations.
3. familiarize data interpolation using several techniques.
4. explore various methods for numerical differentiation and integration.
5. discuss numerical solutions of ordinary and partial differential equations.

Course Content:

Roots of non-linear equations – Bisection method – Fixed points of functions – Fixed point iteration – Regula-Falsi method – Newton-Raphson method – Newton-Raphson method for solution of a pair of non-linear equations – Error analysis for iterative methods.

Solution of system of linear equations – Direct methods – Gauss elimination, Gauss-Jordon and LU-decomposition methods – Pivoting strategies – Iterative methods – Jacobi and Gauss-Seidel methods – Convergence criteria – Eigenvalues and Eigenvectors – Dominant and smallest Eigenvalues/Eigenvectors by power method.

Interpolation – Newton's, Bessel and Stirling's interpolation formulae, divided differences, Lagrange interpolation and Newton's divided difference interpolation – Spline interpolation – Error estimates.

Numerical differentiation – First and second order derivatives by various interpolation formulae – Numerical integration – Newton-Cotes formulae (both simple and composite) with error analysis – Gaussian quadrature formulae.

Solution of first and second order ordinary differential equations – Taylor series method, Euler, modified Euler, Picard's method, Runge - Kutta methods, Milne's and Adam's method – Stability analysis for single-step and multi-step method –Difference formulae for partial derivatives and boundary value problems.

Reference Books:

1. D. Kincaid and W. Cheney, *Numerical Analysis and Mathematics of Scientific Computing*, Brooks/Cole Cengage Learning, 1999.
2. K. Atkinson, *Elementary Numerical Analysis*, John Wiley & Sons, 2004.
3. R. L. Burden and J. D. Faires, *Numerical Analysis*, 9th edition, Brooks/Cole Cengage Learning, 2011.
4. M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Numerical Method: For Scientific and Engineering Computation*, New Age International, 2019.
5. J. H. Mathews and K. K. Fink, *Numerical Methods Using MATLAB*, Pearson, 2004.

Course Outcomes: On completion of the course, students will be able to

1. find the roots of transcendental and polynomial equations.
2. compute the numerical solution of system of linear equations and find dominant eigenvalues of the matrix.
3. use various interpolation methods for the data.
4. find numerical differentiation and numerical integration.
5. compute approximate solution of boundary value problems.

Course Code	MA705
Title of the Course and Type	Probability and Statistics (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. recall the basic probability concepts and introduce some of the special distributions.
2. familiarize two dimensional random variables and various moments.
3. explore the application of law of large numbers and central limit theorem.
4. introduce point estimator and interval estimation for various statistical measures.
5. discuss hypothesis testing for large and small samples.

Course Content:

Definitions of probability – Probability space – Random variables – Probability mass and density functions – Discrete and continuous distributions – Standard and non-standard types.

Mathematical expectation – Generating functions – Probability and moment generating functions – Characteristic function – Two-dimensional random variables – Joint distribution function – Conditional density and expectations – Covariance – Coefficient of correlation – Multiple random variables and its moments.

Chebyshev and Markov inequalities – Convergences in probability – Law of large numbers – Central limit theorem – Applications.

Fundamental concepts in statistics – Measures of location and variability – Population, sample, parameter and statistic – Point and interval estimation – Method of moments, maximum likelihood estimator and properties of estimator – Unbiasedness, consistency and efficiency – Confidence intervals for mean, difference of means and proportions.

Testing of hypothesis – Null and alternate hypothesis – Neymann-Pearson fundamental lemma – Tests for one sample and two sample problems for normal populations – Tests for proportions – Test for small samples – t-test, chi-square test and F-test.

Reference Books:

1. W. Feller, *An Introduction to Probability Theory and its Applications*, 3rd edition, Vol. I and Vol II, Wiley, 2008.
2. K. L. Chung, *A course in Probability Theory*, 3rd edition, Academic Press, 2000.
3. S. M. Ross, *A First Course in Probability*, 10th edition, Pearson Education, 2022.
4. R. V. Hogg, J. W. McKean and A. T. Craig, *Introduction to Mathematical Statistics*, 8th edition, Pearson, 2021.
5. A. M. Mood, F. A. Graybill and D. C. Boes, *Introduction to the Theory of Statistics*, 3rd edition, McGraw Hill, 2017.

Course Outcomes: On completion of the course, students will be able to

1. understand the axiomatic rudiments of modern probability theory and use of random variables as an intrinsic tool for the analysis of random phenomena.
2. characterize multiple input and output system probability models based on single & multiple random variables.
3. evaluate moments, characteristic functions and understand the concept of inequalities and probabilistic limits.
4. compute point estimators and interval estimation for various statistical measures.
5. use basic statistical knowledge in testing of hypotheses on large and small samples.

Course Code	MA706
Title of the Course and Type	R Language (ELR)
Prerequisite	Nil
Credits (L-T-P)	1 – 0 – 1

Course Learning Objectives: The objective of this course is to
1. introduce R programming language.
2. study various data types and data structures in R.
3. explore various control statements and loop structures.
4. provide knowledge about various graphical representations using R.
5. discuss the use of loop functions in computing statistical measures.

Course Content:

Introduction to R – R data types and objects – Reading and writing data (import and export) – Data structures – Vectors, matrices, lists and data frames.

Writing R functions – Control statements – Loops in R – Scoping rules.

Computation of various statistical measures using R.

Creating bar plots, pie charts, box plots and histogram – Curve fitting.

Date and time – Loop functions – Debugging tools.

Reference Books:

1. J. P. Lander, *R for everyone: Advanced Analytics and Graphics*, 2nd edition, Pearson Education, 2018.
2. S. Rakshit, *R for Beginners*, Pearson, McGraw Hill Education, 2017.
3. N. Matloff, *The Art of R Programming: A Tour of Statistical Software Design*, No Starch Press, 2011.
4. P. Dalgaard, *Introductory Statistics with R*, 2nd edition, Springer, 2008.
5. K. Ren, *Learning R Programming*, Packt Publishing Ltd, 2016.

Course Outcomes: On completion of the course, students will be able to
1. explain various data types and data structures in R.
2. write functions using control statements and loops.
3. compute various statistical measures using R.
4. generate various plots for the given data.
5. perform various manipulations with date and time.

Course Code	MA707
Title of the Course and Type	Abstract Algebra (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. introduce the concepts of group actions and direct product of groups.
2. explain the Sylow's theorem and fundamental theorem of finite abelian groups.
3. explore the various types of ideals and integral domains.
4. expose the students to extensions field and its properties.
5. introduce Galois theory and fundamental theorem of Galois theory.

Course Content:

Review of basic group theory – Direct sum and direct product of groups – Group actions – Cayley's theorem – The class equation.

Sylow's theorem and applications – Simple groups – Fundamental theorem of finite abelian groups. Solvable groups.

Review of basic ring theory – Ideals and quotient rings – Prime and maximal ideals – Euclidean domains – Principal ideal domains – Unique factorization domains – Polynomial rings – Irreducibility of polynomials – Ring of Gaussian integers.

Basic theory of field Extensions – Splitting fields – Algebraic and Transcendental extensions – Simple extensions – Separable extensions – Finite fields and related theorems.

Introduction to Galois theory – Fundamental theorem of Galois theory.

Reference Books:

1. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3^{rd} edition, Wiley, 2011.
2. M. Artin, *Algebra*, 2^{nd} edition, Pearson, 2015.
3. I. N. Herstein, *Topics in Algebra*, 2^{nd} edition, Wiley, 2022.
4. J. A. Gallian, *Contemporary Abstract Algebra*, 4^{th} edition, Narosa, 2008.
5. N. Jacobson, *Basic Algebra I and II*, 2^{nd} edition, Dover Publication, 2009.

Course Outcomes: On completion of this course, students will be able to

1. demonstrate a solid understanding of advanced group theory concepts.
2. apply the fundamental theorem of finite abelian groups to solve problems involving group structure.
3. understand and utilize the ideals, factor rings, work within Euclidean domains, principal ideal domains, and unique factorization domains.
4. develop a comprehensive understanding of field extensions, including splitting fields, algebraic and transcendental extensions, simple extensions, separable extensions, and finite fields.
5. comprehend the fundamental theorem of Galois theory and its implications for the correspondence between field extensions and group theory.

Course Code	MA708
Title of the Course and Type	Complex Analysis (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. derive properties of analytic functions and branch of logarithm.
2. examine properties of Mobius transformation and express analytic functions as power series.
3. derive various forms of Cauchy's integral formula/theorems.
4. classify singularities and obtain Laurent's series expansions.
5. discuss Cauchy's residue theorem to evaluate integrals and identify zeros of analytic functions.

Course Content:

Lines and planes in complex plane – Extended complex plane – Spherical representation – Power series – Sequence of functions – Differentiability – Cauchy-Riemann equations – Analytic functions and branch of logarithm.

Mobius transformations – Cross ratios, symmetry and orientation principles – Complex integration and power series representation of analytic functions.

Zeros of analytic functions – Liouville's theorem – Fundamental theorem of algebra – Lucas theorem - Identity theorem and maximum modulus theorem – Index of a closed curve – Cauchy's theorem and integral formula on open subsets of $\mathbb{C}$ – Homotopic version of Cauchy's theorem – Simple connectedness, counting of zeros – Open mapping theorem and Goursat's theorem.

Singularities – Removable singularities and poles – Laurent series and essential singularities – Casorati-Weiestrass theorem.

Cauchy's residue theorem – Evaluation of real integrals – Argument principle – Rouche's theorem - Maximum/minimum modulus theorems and Schwarz lemma.

Reference Books:

1. J. B. Conway, *Functions of One Complex Variables – I*, 2nd edition, Narosa, 2002.
2. D. Ullrich, *Complex Made Simple*, Volume 97, American Mathematical Society, 2008.
3. H. L. Shin and E. Bernard, *Classical Complex Analysis*, Jones and Bartlett, 2011.
4. L. V. Ahlfors, *Complex Analysis*, 3rd edition, McGraw Hill Co., 2017.
5. W. Rudin, *Real and Complex Analysis*, 3rd edition, McGraw-Hill, 2017.

Course Outcomes: On completion of this course, students will be able to

1. analyze properties of analytic functions and construct branch of logarithm.
2. determine properties of Mobius transformation and express analytic functions as power series.
3. prove various forms of Cauchy's integral formula/theorems.
4. find singularities and residues at them as well as obtain Laurent's series expansions.
5. evaluate real integrals using Cauchy's residue theorem and analyze location of zeros of analytic functions.

Course Code	MA709
Title of the Course and Type	Topology (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. introduce various basic concepts of topological spaces.
2. define continuous functions and construction of continuous functions, homeomorphism, metric topology and quotient topology.
3. explain connectedness, components and related theorems.
4. introduce compactness, limit point compactness, local compactness, compactifications and countability axioms.
5. explain all separation axioms, Hausdorff spaces, regular and normal spaces.

Course Content:

Topological spaces – Open sets – Basis for a topology – Sub-basis – Order topology – Finite product topology – Subspace topology – Closed sets – Closure and interior of a set – Limit points of a set – T1, T2 and Hausdorff spaces.

Continuous functions – Homeomorphisms – Constructing continuous functions – Pasting lemma – Arbitrary product topology – Box topology – Metric topology – Quotient topology.

Connected spaces – Connected spaces of the real line – Components and locally connectedness – Path connectedness.

Compact spaces – Limit point compactness – Sequentially compactness – Local compactness – Compactifications – Countability axioms.

Separation axioms – Regular and normal spaces – Urysohn's lemma – Urysohn's metrization theorem – Tietze extension theorem – Tychonoff theorem.

Reference Books:

1. J. R. Munkres, *Topology*, 2^{nd} edition, Pearson Education, 2021.
2. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Mc- Graw Hill education, 2017.
3. C. W. Patty, *Foundations of Topology*, 2^{nd} edition, Jones and Bartlett, 2010.
4. V. V. Prasolov, *Intuitive Topology*, American Mathematical Society, 1995.
5. J.R. Weeks, *The Shape of Space*, 2^{nd} edition, Marcel Dekker, 2002.

Course Outcomes: On completion of this course, students will be able to

1. understand various basic concepts like open sets, basis for a topology, closed sets, product topology, order topology and solve related problems.
2. understand continuous functions and construction of continuous functions, homeomorphism, metric topology, quotient topology.
3. understand connectedness and components and theorems about connectedness, local connectedness and path connectedness and do related problems.
4. understand compactness, limit point compactness, local compactness, compactifications and countability axioms and prove their properties.
5. understand all separation axioms, Hausdorff spaces, Regular, Normal spaces and prove theorems on separation axioms, Urysohn's lemma, Urysohn's metrization theorem, Tietze extension theorem and Tychonoff theorem.

Course Code	MA710
Title of the Course and Type	Partial Differential Equations (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. generate an intuitive understanding of first order partial differential equation describing physical phenomena.
2. familiarize the classification and canonical forms of higher-order partial differential equations.
3. explain concepts and theory of basic methods for solving wave equation.
4. introduce method of separation of variables for diffusion equations.
5. discuss different types of boundary value problems for elliptic equations.

Course Content:

First-order linear partial differential equations – Well-posed problems – Method of characteristics – Integral surfaces passing through a given curve – Surfaces orthogonal to a given system of surfaces – Compatible system of equations – Charpit's method.

Classification of second order partial differential equations – Reduction to canonical form – Adjoint operators.

One-dimensional wave equation – Initial value problem – D'Alembert's solution – Riemann-Volterra solution – Vibrating string – Solution by method of separation of variables – Forced vibrations – Solutions of non-homogeneous equation – Vibration of a circular membrane.

Solution of unsteady one and two-dimensional diffusion equations – Method of separation of variables – Solution in Cartesian, cylindrical and spherical polar coordinates.

Occurrence of the Laplace and Poisson equations – Boundary value problems – Properties of harmonic functions – Separation of variables – Dirichlet and Neumann problems for a rectangle – interior and exterior Dirichlet problem for a circle.

Reference Books:

1. I. N. Sneddon, *Elements of Partial Differential Equations*, McGraw Hill, 1985.
2. L. C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics, 2nd edition, American Mathematical Society, 2010.
3. T. Amarnath, *An Elementary Course in Partial Differential Equations*, Narosa Publishing Company, 1997.
4. F. John, *Partial Differential Equations*, 4th edition, Springer, 1982.
5. Tyn Myint-U and L. Debnath, *Partial Differential Equations for Scientists and Engineers*, 3rd edition, Appleton & Lange, 1989.

Course Outcomes: On completion of the course, students will be able to

1. solve first-order linear and nonlinear partial differential equations using the method of characteristics.
2. recognize and reduce the second-order partial differential equations to its canonical form.
3. model the classical wave equations of mathematical physics and discuss their solutions.
4. identify the classical diffusion equations and discuss their solutions.
5. solve elliptic equations using method of separation of variables.

Course Code	MA711
Title of the Course and Type	Scientific Computing using Python (ELR)
Prerequisite	Nil
Credits (L-T-P)	1– 0 – 1

Course Objectives: The objective of this course is to

1. introduce basics of Python programming.
2. explore various Python modules and their uses.
3. discuss basic mathematical computations using Python.
4. study basic plots and visualizations techniques.
5. introduce good coding practices.

Course Content:

Basics of Python programming – Syntax, variables, data types – Control structures – Loops and conditionals.

Functions and Modules – Writing and using functions – Importing and using Python modules.

Basic arithmetic operations – Using Python for algebraic computations (e.g., solving simple equations) – Introducing sklearn (e.g., linear regression), numpy.

Basic plotting with Matplotlib and seaborn– Visualizing mathematical functions and data.

Writing clean and efficient code – Basic debugging techniques – Documenting code.

Reference Books:

1. J. M. Zelle, *Python programming: An Introduction to Computer Science*, Franklin, Beedle & Associates, 2004.
2. A. B. Downey, *Think Python: How to Think Like a Computer Scientist*, 2nd edition, Green Tea Press, 2012.
3. E. Matthes, *Python crash course: A hands-on, project-based introduction to programming*, no starch press, 2023.
4. D. Mitsotakis, *Computational mathematics: An introduction to numerical analysis and scientific computing with Python*, Chapman and Hall/CRC, 2023.
5. R. Johansson, *Numerical Python: A Practical Techniques Approach for Industry*, Apress, 2015.

Course Outcomes: On completion of the course, students will be able to

1. develop basic proficiency in Python programming
2. understand and use various Python modules.
3. perform basic mathematical computations using Python.
4. create basic plots and visualizations.
5. develop good coding practices and use Python to solve introductory-level mathematical problems.

Course Code	MA712
Title of the Course and Type	Functional Analysis (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. introduce normed linear spaces and Banach spaces.
2. discuss the properties of finite dimensional spaces and bounded linear operators.
3. explain the Hahn-Banach theorem, open mapping theorem and closed graph theorem.
4. introduce various notions used in inner product spaces.
5. teach projection and Riesz representation theorems.

Course Content:

Normed linear spaces – Metric induced by the norm – Cauchy's sequence – Convergence of a sequence – Continuity of vector space operations and norm – Types of convergence of a series – Banach spaces – Finite dimensional normed linear spaces and subspaces – Equivalent norms and their properties.

Riesz's Lemma – Compactness and finite dimension – Quotient space and products of normed linear spaces – Bounded linear transformations – Space of Bounded linear transformations $B[X,Y]$ and its properties – Continuous linear functionals and duals of classical spaces.

The Hahn-Banach extension theorem and its consequences – Baire category theorem – Open mapping theorem and its applications (inverse mapping and closed graph theorems) – Principle of uniform boundedness (Banach-Steinhaus theorem).

Inner product spaces – Norm induced by the inner product – Schwartz inequality – Parallelogram identity – Hilbert spaces – Relation between Banach and Hilbert spaces – Orthogonal complement of a subspace – Orthogonal decomposition – Orthogonal and orthonormal systems – Bessel's inequality – Parseval's identity.

Characterizations of complete orthonormal systems – Riesz-Fischer theorem – Closest point in a closed convex subset – Projection theorem – Riesz representation theorem – Reflexivity of the Hilbert space.

Reference Books:

1. B. Bollobas, *Linear Analysis*, 2nd edition, Cambridge University Press, 1999.
2. J. B. Conway, *A Course in Functional Analysis*, Graduate Texts in Mathematics, 2nd edition, Springer, 2010.
3. E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, 2007.
4. G. F. Simmons, *Introduction to Topology and Modern Analysis*, 2nd revised edition, Medtech Science Press, 2024.
5. A. E. Taylor and D. C. Lay, *Introduction to Functional Analysis*, 2nd edition, Wiley, 1980.

Course Outcomes: On completion of this course, students will be able to

1. understand various concepts of normed linear spaces and construct new spaces from old ones.
2. find duals of standard spaces and prove fundamental results about bounded linear functionals.
3. develop a comprehensive understanding of the major theorems of functional analysis and their applications.
4. evaluate orthogonal decomposition of Hilbert spaces and prove several other results.
5. comprehend projection theorem and Riesz representation theorem.

Course Code	MA713
Title of the Course and Type	Operations Research (Program Core)
Prerequisite	Nil
Credits (L-T-P)	3 – 1 – 0

Course Learning Objectives: The objective of this course is to

1. discuss various real-life problems as operations research models and methodologies to solve them.
2. introduce linear programming, transportation and assignment problems to discuss their solution methodologies.
3. study duality theory and sensitivity analysis in linear programming.
4. explore dynamic programming problem and its applications.
5. study non-linear problems and various solution techniques.

Course Content:

Introduction of different models in operations research – Linear programming problems – Simplex method – Big-*M* method – Two-phase method –Degeneracy and cycling – Unbounded solutions – Alternative optima.

Revised simplex method – Sensitivity analysis – Parametric programming – Dual-linear programs – Duality theorems – Dual-simplex method.

Transportation problems – Finding an initial basic feasible solution – Optimality condition – MODI method – Degeneracy – Assignment problems – Hungarian method.

Introduction to dynamic programming – Principle of optimality – Forward and backward recursions – Discrete dynamic programming – Continuous dynamic programming and applications.

Non-linear programming problem – Kuhn-Tucker conditions – Wolfe's and Beale's methods for solving quadratic programming problem.

Reference Books:

1. A. Ravindran, D. T. Phillips and J. J. Solberg, *Operations Research: Principles and Practice*, 2nd edition, Wiley, 2007.
2. H. A. Taha, *Operations Research – An Introduction*, 10th edition, Pearson Education, 2019.
3. F. S. Hillier, G. J. Lieberman, B. Nag and P. Basu, *Introduction to Operations Research*, 11th edition, McGraw Hill Education, 2021.
4. V. L. Mote and T. Madhavan, *Operations Research*, 1st edition, Wiley, 2016.
5. S. S. Rao, *Optimization Theory and Applications*, 4th edition, Wiley, 2013.

Course Outcomes: On completion of this course, students will be able to

1. solve linear programming problem using simplex, Big-M and two-phase methods.
2. perform sensitivity analysis and apply duality theory.
3. identify and evaluate transportation and assignment problems.
4. utilize the solutions techniques for dynamic problems.
5. analyze and solve nonlinear programming problems.

Course Code	MA714
Title of the Course and Type	Mathematical Software Lab (ELR)
Prerequisite	Nil
Credits (L-T-P)	1 – 0 – 1

Course Objectives: The objective of this course is to

1. introduce scientific computing software available to solve mathematical problem.
2. practice basic operators and functions available in SCILAB.
3. write SCILAB code for mathematical algorithms.
4. discuss fundamentals of typesetting in LaTeX and explore TikZ package to draw basic elements.
5. explore formatting research articles, books, theses and creating slides using LaTeX.

Course Content:

Introduction to SCILAB environment – Different data types and constants – Vector and matrix operations – Polynomials.

SCILAB functions – Built-in functions – User-defined functions – Relational and logical operators – While and FOR loop – details of loop operations – Break and continue statements – Nested loops.

File handling in SCILAB – Introduction to plotting – 2D and 3D plotting – Developing the skills of writing a program for solving differential equations.

Running LaTeX – Document class – Creating lists – Rows and columns – Keeping tabs – Simple tables – Complex tables with merged rows/columns – Introduction to bibliographic environment – Basic commands – Multiple citation – BibTeX programme – BibTeX style files – Creating a bibliographic database. TikZ package – Basic shapes – Grids – Color filling – Axes – Generating TikZ code from GeoGebra.

Typesetting mathematics – Superscripts and subscripts – Different mathematical symbols (roots, fraction) – Matrices, numbered equations, single equations, group of equations, array environment – Typesetting theorems. Preparing research articles, books, thesis – Preparing slides using beamer – Inserting pictures – Letters and notes.

Reference Books:

1. S. Nagar, *Introduction to SCILAB: For Engineers and Scientist*, Apress, 2017
2. T. Sheth, *Scilab: A Practical Introduction to Programming and Problem Solving*, CreateSpace Independent Publishing Platform, 2016.
3. L. Lamport. *LaTeX: A Document Preparation System, User's Guide and Reference Manual*, 2nd edition, Addison-Wesley, 1994.
4. D. F. Griffiths and D. J. Higham, *Learning LaTeX*, 2nd edition, SIAM, 2016.
5. H.C. Johnson, *Mastering LaTeX: Advanced Drawing with Tikz*, Telephasic Workshop, 2024.

Course Outcomes: On completion of the course, the students will be able to

1. perform basic operations and symbolic calculations in SCILAB.
2. understand and use in-built functions.
3. create SCILAB codes to solve problem in ordinary differential equations, linear algebra, trigonometry.
4. create basic shapes using TikZ and typeset mathematical theorems and proofs using LaTeX.
5. format research articles, books, theses and create slides for presentations using the beamer class.

Course Code	MA721
Title of the Course and Type	Integral Transforms (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. teach Laplace transforms and related theorems.
2. define and analyze periodic functions using the Fourier series, including half-range and complex forms.
3. discuss Fourier transforms for heat equations and problems with various boundary conditions.
4. explore applications of Laplace and Fourier transforms.
5. introduce Z-transforms to formulate and solve difference equations.

Course Content:

Laplace Transforms – Transforms of elementary functions – Properties – Differentiation and integration of transforms – Periodic functions – Initial and final value theorems – Inverse Laplace transforms – Convolution theorem – Error function – Transforms involving Bessel functions.

Fourier series – Dirichlet's conditions – General Fourier series – Half-range sine series – Half-range cosine series – Complex form of Fourier series – Parseval's identity – Harmonic analysis.

Fourier transforms – Fourier integral representation – Fourier transform pairs – Properties – Fourier sine and cosine transforms – Transforms and inverse transforms of elementary functions – Convolution theorem – Transforms of derivatives.

Application of Laplace transforms – Evaluation of integrals – Solution of linear ordinary differential equations – Applications of Fourier transforms – Heat equation on infinite and semi-infinite line – Potential problems in half-plane.

Z-transforms and difference equations – Elementary properties – Inverse Z-transform (using partial fraction and residues) – Convolution theorem – Formation of difference equations – Solution of difference equations using Z-transforms.

Reference Books:

1. L.C. Andrews and B.K. Shivamoggi, *Integral Transforms for Engineers*, SPIE press, 1999.
2. I.N. Sneddon, *Fourier Transforms,* Courier Corporation, 1995.
3. L.C. Andrews and B.K. Shivamoggi, *Integral Transforms for Engineers and Applied Mathematicians*, Mac Millan Publishing Co., 1988.
4. L. Debnath and D. Bhatta, *Integral Transforms and Their Applications*, CRC Press, 2016
5. A. C. Grove, *An Introduction to the Laplace transform and the Z-transform*, Prentice Hall, 1991.

Course Outcomes: On completion of the course, students will be able to
1. compute Laplace transforms of various functions and their inverses.
2. use the Fourier series to decompose periodic functions and solve related problems.
3. find Fourier transforms of elementary functions.
4. apply transform techniques to solve ordinary and partial differential equations.
5. employ Z-transforms for solving difference equations.

Course Code	MA722
Title of the Course and Type	Graph Theory (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce fundamentals of graph theory.
2. discuss problems related to matching.
3. discuss the concepts of connectivity and its applications.
4. explore the notions of vertex coloring and planar graphs.
5. identify Euler graphs and learn finding approximate solutions of travelling salesman problem**.**

Course Content:

Definition of graphs – Matrices and isomorphism – Connection in graphs – Bipartite graphs – Eulerian circuits – Vertex degrees and counting – Graphic sequences – Directed graphs – Eulerian digraphs – Orientations and tournaments – Trees – Basic properties – Distance in trees and graphs – Spanning trees and enumeration of trees – Cayley's formula – Minimum spanning tree – Shortest path.

Maximum matchings – Hall's matching condition – Min-Max theorems – Independent sets and covers – Dominating sets – Maximum bipartite matching – Weighted bipartite matching – Tutte's 1-factor theorem.

Connectivity – Edge-connectivity – Blocks – 2-connected graphs – Connectivity of digraphs – k-connected and k-edge connected graphs – Menger's theorem and applications – Network flow problems.

Vertex colorings and upper bounds – Brook's theorem – Structure of k-chromatic graphs – Turan's theorem – Planar graphs – Embedding's and Euler's formula – Characterization of planar graphs – Kuratowski's theorem (without proof).

Characterization of Euler graphs – Necessary/sufficient conditions for the existence of Hamiltonian cycles – Fleury's algorithm for Eulerian trails – Chinese-postman problem – Approximate solutions of travelling salesman problem**.**

Reference Books:

1. D. B. West. *Introduction to Graph Theory*, 2nd edition, Pearson Education, 2015.
2. R. J. Trudeau. *Introduction to Graph Theory*, 2nd edition, Dover publication, 1993.
3. J. A. Bondy and U.S.R. Murty. *Graph Theory*, Graduate Texts in Mathematics, Vol. 244, Springer, 2008.
4. R. Diestel. *Graph Theory,* Graduate Texts in Mathematics, vol. 173, 5th edition, Springer, 2017.
5. R. Balakrishnan and K. Ranganathan. *A Text Book of Graph Theory*, 2nd edition, Springer, 2012.

Course Outcomes: On completion of the course, students will be able to

1. find minimum spanning trees and shortest paths in graphs.
2. apply Tutte's 1-factor theorem to solve problems related to bipartite matchings.
3. solve connectivity problems in graphs and digraphs.
4. explore Turan's theorem to solve graph coloring problems and characterize planar graphs.
5. evaluate Eulerian trails and apply approximate solutions for the travelling salesman problem in practical scenarios.

Course Code	MA723
Title of the Course	Mathematics of Machine Learning
Prerequisite	Nil
Credits (L-T-P)	3- 0 - 0

Course Learning Objectives: The objective of this course is to

1. introduce fundamental probability and statistical concepts to machine learning problems.
2. discuss optimization problems using techniques such as gradient descent and its variants.
3. teach linear models, including linear and logistic regression, with regularization methods.
4. explore bias-variance tradeoff, perform model selection, and apply cross-validation techniques for model validation.
5. study neural networks and deep learning architectures to real-world applications like image recognition and natural language processing.

Course Content

Basic probability concepts – Random variables and distributions – Expectation and variance – Common probability distributions – Bayes' theorem and Bayesian inference – Applications in machine learning: Naive Bayes classifier.

Introduction to optimization problems – Gradient descent and its variants – Convex optimization, stochastic gradient descent – Applications in machine learning: Training neural networks – Support Vector Machines (SVMs).

Linear regression – Logistic regression – Regularization techniques: Ridge and Lasso regression – Decision tree/random forest classifiers, xgboost classifier – Applications in machine learning: Predictive modeling, binary classification.

Bias-variance tradeoff – Overfitting and underfitting – Model selection and validation – Cross-validation techniques – Applications in machine learning: Ensuring model generalizability.

Introduction to neural networks – Forward and backward propagation – Activation functions – Loss functions and optimization – Deep learning architectures: Convolutional Neural Networks (CNNs) – Recurrent Neural Networks (RNNs) – Applications in machine learning: Image recognition, natural language processing.

Reference Book:

1. C. M. Bishop, *Pattern Recognition and Machine Learning*, Springer, 2006.
2. S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.
3. G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, 2012.
4. T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, Springer, 2009.
5. I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*, MIT Press, 2016.

Course Learning Outcomes: On completion of the course, students will be able to

1. use probability concepts and Bayesian inference for machine learning.
2. implement gradient descent and convex optimization to train machine learning models.
3. create and assess linear and logistic regression models with regularization for prediction and classification.
4. manage bias-variance tradeoff and use model selection and cross-validation to improve model performance.
5. build and optimize deep learning models, including CNNs and RNNs, for tasks such as image recognition and natural language processing.

Course Code	MA731
Title of the Course and Type	Fluid Dynamics (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce the notion of a fluid, approaches and framework to deal with the motion of a fluid.
2. discuss equations of motion of inviscid fluids and related concepts using examples.
3. explore stress-strain relations and study the equations of motion of a viscous fluid.
4. introduce dimensionless ways of writing equations using the foundations of dimensional analysis.
5. discuss some exact solutions and the notion of boundary layer theory.

Course Content:

Concept of a fluid – Fluid as a continuum – Dimensions and units – Basic properties of fluids – Viscous and inviscid fluids – Newtonian and non-Newtonian fluids – Some important types of flows – Eulerian and Lagrangian coordinates – Material derivative – Acceleration of a fluid – Path lines and streamlines – Linear strain rate – Shear strain rate – Velocity potential function– Vorticity vector.

Equation of continuity – Euler's equation of motion – Bernoulli's equation and applications – Stream function – Potential lines – Complex potential – Two and three dimensional potential flows – Sources and sinks – Flow due to doublet.

Relation between stress and rate of deformation – Stoke's hypothesis – Navier-Stokes equations – Vorticity transport equation – Boundary conditions for the basic equations.

Dimensional homogeneity and similarity – Some dimensionless numbers and coefficients employed in the study of fluid flows – Technique of dimensional analysis – Rayleigh's technique – Buckingham π-theorem and applications.

Some exact solutions of Navier-Stokes equations – Parallel flows – Stagnation in a plane flow – Concept of boundary layer – Boundary layer along a flat plate.

Reference Books:

1. F. M. White, *Fluid Mechanics*, McGraw-Hill, 2011.
2. F. Chorlton, *Textbook of Fluid Dynamics*, CBS Publishers, 2016.
3. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press, 2000.
4. I. G. Currie, *Fundamental Mechanics of Fluids*, CRC press, 2002.
5. H. Schlichting, and G. Klaus, *Boundary-layer Theory*, Springer, 2016.

Course Outcomes: On completion of the course, students will be able to

1. understand physical concepts involved in fluid motion.
2. model some two and three dimensional inviscid flow problems through source/sink doublet.
3. derive equations of motion for viscous fluids.
4. use the techniques of dimensional analysis to reduce the complexity of a given flow problem.
5. find exact solution of some fluid flow problems and interpret results physically.

Course Code	MA732
Title of the Course and Type	Integral Equations and Calculus of Variations (Program Elective)
Prerequisites	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce a comprehensive understanding of integral equations, basic concepts and classifications.
2. famililarize Volterra and Fredholm integral equations and their solution using method of successive approximations.
3. train the utilization of iterated, resolvent kernels for solving integral equations and construct Green's function for boundary value problems.
4. explore the Euler-Lagrange equations in solving variational problems.
5. discuss the approximate methods to solve variational problems and boundary value problems.

Course Content:

Basic concepts of integral equations – Volterra and Fredholm equation with 1st and 2nd kinds – Relationship between linear differential equations with Volterra and Fredholm types.

Resolvent kernel – Method of successive approximations – Convolution type equations – Abel's integral equation – Method of Fredholm determinants.

Iterated kernels – Integral equations with degenerate kernels – Eigenvalues and eigenfunctions – Fredholm alternative – Green's function for boundary value problems – Singular integral equations.

Calculus of Variations – Euler-Lagrange equations – Degenerate Euler equations – Natural boundary conditions – Transversality conditions – Simple applications of variational principle – Sufficient conditions for extremum.

Variational formulation of boundary value problems – Minimum of quadratic functional – Approximate methods – Galerkin's method – Weighted-residual methods – Collocation methods – Variational methods for time dependent problems.

Reference Books:

1. D. Porter and D. S. G. Stirling. *Integral Equations: A Practical Treatment, from Spectral Theory to Applications*, Cambridge texts in Applied Mathematics, 1990.
2. R. P. Kanwal, *Linear Integral Equations: Theory and Technique*, 2nd edition, Springer, 2012.
3. A. M. Wazwaz, *Linear and Non-linear Integral Equations: Methods and Applications*, 1st edition, Springer, 2011.
4. R. Weinstock, *Calculus of Variations (with Applications to Physics and Engineering)*, revised edition, Dover Publications, 1974.
5. I. M. Gelfand, and S. V. Fomin, *Calculus of Variations*, revised edition, Dover Publications, 2000.

Course Outcomes: On completion of the course, students will be able to

1. explain basics of Volterra and Fredholm integral equations and their relationship with ordinary differential equations.
2. apply the method of successive approximations and resolvent kernels to solve integral equations.
3. construct Green's functions for boundary value problems.
4. apply Euler-Lagrange equations to solve natural boundary and transversality conditions problems, and analyze sufficient conditions for extremum.
5. design and evaluate variational methods for time-dependent problems using approximation techniques such as Galerkin and weighted-residual methods.

Course Code	MA733
Title of the Course and Type	Measure Theory (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce the foundational concepts of measure theory and Lebesgue outer measure.
2. explore Lebesgue integral including its properties and key convergence theorems.
3. compare Riemann and Lebesgue integration and understand the conditions for which functions are integrable in both senses.
4. investigate functions of bounded variation and their relation with differentiation and integration.
5. explore the concepts of signed measures, Hahn-Jordan decompositions and Radon-Nikodym theorem.

Course Content:

Ring and algebra of sets, sigma-algebras – Borel algebra and Borel sets – Lebesgue outer measure on real line – Countable sub-additivity, measurable set, sigma-algebra structure of measurable sets – Countable additivity of Lebesgue measure on real line – Cantor set – Construction of a non-measurable subset of [0, 1] – Measurable functions – Approximation of measurable functions – Egorov's theorem – Lusin's theorem.

Lebesgue integral of non-negative measurable functions – Integrable functions and Lebesgue integral of integrable functions – Linearity – Monotone convergence theorem – Fatou's lemma – Dominated convergence theorem.

Comparison of Riemann and Lebesgue integration – Lebesgue integrability of Riemann integrable functions – Characterization of Riemann integrable functions – Improper Riemann integrals and their Lebesgue integrals.

Functions of bounded variation – Indefinite integrals of Lebesgue integrable functions on $[a, b]$ – Vitali's lemma – Almost everywhere differentiability of monotonically increasing functions – Absolutely continuous functions and their properties – Absolute continuity of indefinite integral of Lebesgue integrable functions – Differentiation of indefinite integrals – Characterization of absolutely continuous functions as indefinite integrals.

Completeness of $L_p\ (\mu)$ spaces – Signed measures – Hahn and Jordan decompositions – Radon-Nikodym theorem (without proof).

Reference Books:

1. G. De Barra, *Measure and Integration*, 1st edition, New Age International, 2013.
2. P. R. Halmos, *Measure Theory*, Graduate Texts in Mathematics vol. 18, Springer, 2013.
3. E. M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*, Princeton Lectures in Analysis, Vol. III, 2005.
4. H. L. Royden, P. M. Fitzpatrick, *Real Analysis*, 4th edition, Pearson Education, 2011.
5. W. Rudin, *Real and Complex Analysis*, 3rd edition, McGraw-Hill International Editions, 2017.

Course Outcomes: On completion of the course, students will be able to

1. apply the concepts of rings, algebras, sigma-algebras, and Borel sets in the context of measure theory.
2. compute Lebesgue integral for non-negative measurable and integrable functions.
3. compare and contrast Riemann and Lebesgue integration.
4. analyse functions of bounded variation, their indefinite integrals and apply Vitali's lemma.
5. verify the completeness of standard measurable spaces.

Course Code	MA734
Title of the Course and Type	Optimization Techniques (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce convex sets, convex functions and extreme points and extreme directions of polyhedral sets.
2. study first-order and second-order optimality conditions for unconstrained optimization problems.
3. learn Fitz John and Kuhn-Tucker conditions for constrained optimization problems.
4. discuss methods to solve unconstrained and constrained optimization problems.
5. introduce the Lagrangian dual problem and to study properties of the dual function.

Course Content:

Convex sets – Convex Hulls – Caratheodory theorem – Separation theorems and Farkas' lemma – Convex cones – Polyhedral sets – Extreme points – Linear programming.

Convex optimization – Convex functions – Epigraph – Directional derivative – Subgradients – Differentiable convex functions – Hessian matrix – Local and global optimality.

Unconstrained optimization – Optimality conditions – First-order and second-order conditions – Method of steepest descent – Newton's method – Conjugate gradient method.

Constrained optimization – Problems with equality and inequality constraints – Method of Lagrange multipliers – Fitz John conditions – Karush-Kuhn-Tucker conditions – Frank-Wolfe's method – Projected gradient methods – Penalty methods.

Lagrangian duality – Geometric interpretation – Duality theorems – Saddle point optimality – Duality for linear and quadratic optimization problems – Multi objective optimization.

Reference Books:

1. M. S. Bazaraa, H. D. Sherali and C. M. Shetty, *Nonlinear Programming Theory and Algorithms*, John Wiley, 2013.
2. E. K. P. Chong and S. H. Zak, *An Introduction to Optimization*, John Wiley, 2010.
3. N. Andreasson, A. Evgrafov and M. Patriksson, *An Introduction to Continuous optimization*, Overseas Press, 2006.
4. S. Chandra, Jeyadeva and A. Mehra, *Numerical Optimization with Applications*, Narosa, 2009.
5. S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2009

Course Outcomes: On completion of the course, students will be able to

1. analyze convex sets and convex functions and to compute subgradients of a convex function.
2. verify optimality conditions of an unconstrained optimization problem and to solve it using steepest descent, Newton's and conjugate gradient methods.
3. find an optimum solution to a constrained optimization problem using Karush-Kuhn-Tucker and Fritz John conditions.
4. determine an optimal solution of constrained optimization problem using Frank-Wolfe's, projected gradient and penalty methods.
5. write Lagrangian dual problem to a non-linear programming and apply duality results to solve it.

Course Code	MA735
Title of the Course and Type	Stochastic Processes (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. introduce the basic concepts of stochastic processes and their classification.
2. explain renewal processes with examples.
3. describe Markov chain and its applications.
4. illustrate the discrete time Markov chain with examples.
5. familiarize the procedure for computing mean first passage times.

Course Content:
Stochastic processes – Definition and examples – Classification of stochastic processes with illustrations.

Bernoulli process – Poisson process – Renewal processes.

Markov chains – Introduction and examples – Chapman-Kolmogorov equations – Classification of states – Limit theorems – Transitions among classes – Gambler's ruin problem – Mean time in transient states – Branching processes – Applications of Markov chain – Time-reversible Markov chains – Semi-Markov processes.

Discrete-time Markov chains (DTMC) – Basic concepts, key definitions and results – Computation of the steady-state probability vector of DTMC – Absorbing and Taboo probabilities in DTMC.

Continuous-time Markov chain (CTMC) – Basic concepts, key definitions and results – Computation of exponential matrix, limiting probabilities of CTMC and mean first passage times – Semi-Markov and Markov renewal processes.

Reference Books:

1. S. M. Ross. *Stochastic Processes*, 2nd edition, John Wiley and Sons, 2008.
2. Z. Brzezniak and T. Zastawniak. *Basic Stochastic Processes: A Course through Exercises*, 3rd edition, Springer, 2000.
3. J. Medhi. *Stochastic Processes*, 5th edition, New Age Publishers, 2020.
4. S. Karlin and H.M. Taylor. *A First Course in Stochastic Processes*, 2nd edition, Academic Press, 2011.
5. J. F. Shortle, J. M. Thompson, D. Gross and C. M. Harris, *Fundamentals of Queueing Theory*, 5th edition, John Wiley and Sons, 2017.

Course Outcomes: On completion of the course, students will be able to
1. understand the basic concepts and classifications of stochastic processes.
2. explain the various renewal processes.
3. describe Markov chains and classification of states.
4. analyze discrete-time Markov chain.
5. compute the mean first passage time for a CTMC.

Course Code	MA741
Title of the Course and Type	Numerical Solution of Differential Equations (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. give an understanding of numerical methods for solving boundary value problems in ordinary differential equations.
2. introduce finite difference methods for solving one dimensional and two dimensional parabolic partial differential equations.
3. study hyperbolic equations and various numerical methods of solving them.
4. discuss finite difference methods for solving Laplace and Poisson equations in cartesian and polar coordinates.
5. familiarize with basic concepts of finite element analysis.

Course Content:

Ordinary differential equations – Boundary-value problems – Shooting method and finite difference methods – Convergence analysis.

Parabolic equation – One dimensional parabolic equation – Explicit and implicit finite difference scheme – Stability and convergence of difference scheme – Two dimensional parabolic equations – A.D.I. methods with error analysis.

Hyperbolic equations – First order quasi-linear equations and characteristics – Numerical integration along a characteristic – Lax-Wendroff explicit method – Second order quasi-linear hyperbolic equation – Characteristics – Solution by method of characteristics.

Elliptic equations – Solution of Laplace and Poisson equations in a rectangular region – Finite difference in polar coordinate formulas for derivatives near a curved boundary when using a square mesh – Discretization error – Mixed boundary value problems.

Finite Element Method – Types of integral formulations – One and two dimensional elements – Galerkin formulation – Application to Dirichlet and Neumann problems.

Reference Books:

1. G. Evans, J. Blackledge and P. Yardley. *Numerical Methods for Partial Differential Equations*, 1st edition, Springer, 2012
2. K. W. Morton and D. F. Mayers. *Numerical Solution of Partial Differential Equations: An Introduction*, 4th edition, Cambridge University Press, 2005
3. J. A. Trangenstein, *Numerical Solution of Elliptic and Parabolic Partial Differential Equations*, 2nd edition, Cambridge University Press, 2013.
4. K. E. Atkinson, W. Han and D. E. Stewart. *Numerical Solution of Ordinary Differential Equations*,1st edition, John Wiley & Sons, 2009.
5. J. N. Reddy, *An Introduction to Nonlinear Finite Element Analysis: With Applications to Heat Transfer*, 1st edition, Oxford University Press, 2015.

Course Outcomes: On completion of the course, students will be able to

1. apply finite difference method to solve boundary value problem in ordinary differential equations.
2. design finite difference numerical scheme for parabolic equations and solve numerically.
3. solve hyperbolic equations through method of characteristics and Lax-Wendroff method.
4. find numerical approximations to the solution of Laplace and Poisson equations in Cartesian and polar coordinates.
5. construct integral formulations for the given boundary value problem and solve them using FEM.

Course Code	MA742
Title of the Course and Type	Operator Theory (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. provide an understanding of dual spaces and their representations in various normed spaces.
2. explore the properties and applications of various operators in Hilbert spaces.
3. analyse compact operators, integral operators and Hilbert-Schmidt operators in Banach and Hilbert spaces.
4. understand spectral results for operators on Banach and Hilbert spaces.
5. examine the elementary properties of spectral theory for operators on Banach spaces.

Course Content:

Dual space considerations – Representation of duals of the spaces c_{00} with p-norms, c_0 and c with supremum-norm, l_p, $C[a,b]$ and L_p – Reflexivity, weak and weak* convergences.

Adjoint, self-adjoint, normal and unitary operators on Hilbert spaces and their properties – Projection operators on Banach spaces and Hilbert spaces – Eigenvalues, eigenvectors and eigenspaces – Invariant spaces – Spectral theorem on finite dimensional Hilbert spaces.

Operators on Banach and Hilbert spaces – Compact operators and its properties – Integral operators as compact operators – Adjoint of operators between Hilbert spaces – Self-adjoint, normal and unitary operators – Numerical range and numerical radius – Hilbert-Schmidt operators.

Elementary properties of spectral theory for operators on Banach space – Ideals and quotients – Spectrum, spectrum of a linear operator, spectral theory of a compact self-adjoint and compact normal operators.

Spectral results for Banach and Hilbert space operators – Eigen spectrum, Approximate eigen spectrum – Spectrum and resolvent – Spectral radius formula – Spectral mapping theorem – Riesz-Schauder theory – Spectral results for normal, self-adjoint and unitary operators – Functions of self-adjoint operators.

Reference Books:

1. J. B. Conway, *A Course in Functional Analysis*, *Graduate studies in Mathematics*, 2nd edition, Springer, 2007.
2. J. B. Conway, *A Course in Operator Theory*, *Graduate Studies in Mathematics*, American Mathematical Society, 2000.
3. R. G. Douglas, *Banach Algebra Techniques in Operator Theory*, Graduate Texts in Mathematics, 2nd edition, Springer, 1998.
4. E. Kreyszig, *Introduction to Functional Analysis with Applications*, Wiley, 2007.
5. E. Zeidler, *Applied Functional Analysis: Applications to Mathematical Physics*, Applied Mathematical Sciences vol. 108, Springer, 2012.

Course Outcomes: On completion of the course, students will be able to
1. represent the duals of various normed spaces and use the concepts of reflexivity and weak/weak* convergences.
2. apply the properties of adjoint, self-adjoint, normal, and unitary operators in Hilbert spaces.
3. describe projection operators in Banach and Hilbert spaces.
4. solve problems involving eigenvalues, eigenvectors, eigenspaces and invariant spaces in finite-dimensional Hilbert spaces.
5. apply spectral theory to Banach and Hilbert space operators.

Course Code	MA743
Title of the Course and Type	Introduction to Fuzzy Mathematics and its Applications (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce the types of fuzzy sets, α-cuts and its properties and extension of functions.
2. define different operations (fuzzy complements, fuzzy intersections, and fuzzy unions) on fuzzy sets.
3. teach fuzzy relations and introduce different types of fuzzy relations and their applications.
4. explain applications of fuzzy relations, fuzzy c-means methods to clustering of data (information) in engineering problems.
5. teach the concepts of fuzzy decision-making methods in engineering and management problems.

Course Content:

Fuzzy sets – introduction, basic types and concepts – Additional properties of α-cuts – Representation of fuzzy sets – Extension principles.

Type of operators on fuzzy sets and fuzzy complements – Fuzzy intersection and fuzzy unions – Combination of operations.

Fuzzy numbers and arithmetic operations on intervals – Arithmetic operations on fuzzy numbers – Fuzzy equations and relations – Binary fuzzy relations and binary relations on a single set – Fuzzy equivalence relations.

Classification by equivalence relations – Crisp relations – Fuzzy relations – Cluster Analysis – Cluster Validity – c-means Clustering – Hard c-means (HCM) – Fuzzy c-Means (FCM).

Fuzzy decision making – Introduction – Conversion of linguistic variables to fuzzy numbers – Individual decision making – Multi-person decision making – Multi-criteria decision making – Fuzzy ranking methods.

Reference Books:

1. G. J. Klir, Bo Yuan, *Fuzzy Sets and Fuzzy Logic - Theory and Applications*, Pearson, 2015.
2. T. J. Ross, *Fuzzy Sets and Fuzzy Logic with Engineering applications*, Wiley India 2021.
3. H. J. Zimmermann, *Fuzzy Set theory and its applications*, Springer, 2014.
4. S. J. Chen and C. L. Hwang, *Fuzzy Multiple Attributes Decision Making*, Springer, 1992.
5. G. Chen and T. T. Pham, *Introduction to Fuzzy Sets, Fuzzy Logic and Fuzzy Control Systems,* CRC Press, 2000.

Course Outcomes: On completion of the course, students will be able to

1. analyze different types of fuzzy sets, α-cuts and its properties and extension of functions.
2. apply the operations (fuzzy complements, fuzzy intersections, and fuzzy unions) on fuzzy sets.
3. create the fuzzy relations and identify the different types of fuzzy relations and their applications.
4. implement fuzzy relations, fuzzy c-means methods to clustering of data (information) in engineering problems.
5. use the concepts of fuzzy decision-making methods in engineering and management problems.

Course Code	MA744
Title of the Course and Type	Introduction to Singularly Perturbed Differential Equations (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. introduce the mathematical preliminaries of asymptotic expansions.
2. discuss the asymptotic solution of the singular perturbation problems.
3. explore the boundary layer behaviour of the singular perturbation problems.
4. discuss the existence and stability of the solutions of singular perturbation problems.
5. introduce Wentzel–Kramers–Brillouin (WKB) method for singular perturbation problems.

Course Content:

Mathematical Preliminaries – Little-oh, Big-oh, asymptotically equal to or behaves like notation – Asymptotic sequence and expansions – Convergent series versus divergent series – Asymptotic expansions with parameter – Uniformity or breakdown – Overlap regions – Matching principle – Matching with logarithmic terms – Composite expansion.

Introductory application – Roots of equations – Integration of functions represented by asymptotic expansions – Ordinary differential equations – Regular problems, simple singular problems, scaling of differential equations and equations which exhibit a boundary layer behaviour.

Motivation for the study of singular perturbation problem (SPP) – Asymptotic expansion and approximation – Asymptotic solution of algebraic and transcendental equations – Regular and singular perturbations for first and second-order ordinary differential equations – Physical examples.

Two-point boundary-value problems – Boundary layer – Exponential and cusp layers – Matched asymptotic expansions – Composite asymptotic expansions – WKB method – Conditions for validity of the WKB approximation – Patched asymptotic approximations – WKB solution of inhomogeneous ordinary differential equations.

Boundary layers and transition layers – Method of multiple scales – Nearly linear oscillations –nonlinear oscillators – Applications to classical ordinary differential equations – WKB method for slowly varying oscillations – Turning point problem – Applications to partial equations – Limitation on the use of the method of multiple scales – Boundary layer problems – Some physical applications of singular perturbation problems.

Reference Books:

1. C. M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, 1st edition, Springer, 2010.
2. R. E. O'Malley, *Singular Perturbation Methods for Ordinary Differential Equations*, 1st edition, Springer, 1991.
3. R. C. Mc Owen, *Partial Differential Equations – Methods and Applications*, Pearson Education, 2004.
4. E. A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover Publications, 1989.
5. A. H. Nayfeh, *Introduction to Perturbation Techniques*, Wiley, 2014.

Course Outcomes: On completion of the course, students will be able to
1. determine breakdown points for asymptotic expansions.
2. derive the asymptotic approximations for solutions of singular perturbation problems.
3. deduce the boundary layer behavior of solution of singular perturbation problems.
4. use matched asymptotic method to solve two -point boundary value problems.
5. apply the WKB method to solve singular perturbation problems.

Course Code	MA745
Title of the Course and Type	Matrix Analysis (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. study partitioned matrices and Jordan canonical and singular value decompositions of matrices.
2. investigate Hadamard and Kronecker products and related matrix inequalities.
3. study characterizations and properties of positive semidefinite matrices.
4. discuss nonnegative matrices and Perron-Frobenius Theorem.
5. introduce different classes of matrices and study their relationship with complementarity problems and stability analysis.

Course Content:

Review of basic matrix theory – Determinant and inverse of partitioned matrices – Annihilating polynomial of matrices – Minimal polynomial and diagonalization of matrices – Jordan canonical forms – Spectral theorem – Singular value decomposition.

Positive semidefinite matrices – Characterization and properties – Simultaneous diagonalization – Schur Complement – Fischer and Hadamard inequalities – Kronecker and Hadamard products.

Nonnegative matrices – Irreducible matrices – Perron-Frobenius Theorem – Matrices with positive principal minors (P- matrices) – Sign reversal properties – Global uniqueness of linear complementarity problems.

Matrices with nonnegative principal minors – Sufficient matrices – Nondegenerate matrices – Copositive matrices – Semimonotone matrices – Completely Q-matrices.

Z-matrices and least-element Theory – M-matrices – Characterizations – Positivity of principal minors – Positive stability – Inverse-positivity and splittings – Stieltjes matrices.

Reference Books:

1. F. Zhang, *Matrix Theory - Basic Results and Techniques*, Springer, 2010.
2. R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, 2012.
3. A. Berman and R.J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, SIAM, 1994.
4. R. Bellman, *Introduction to Matrix Analysis*, 2nd edition, SIAM, 1997
5. R.W. Cottle, J-S. Pang and R. E. Stone, *The Linear Complementarity Problem*, SIAM, 2009.

Course Outcomes: On completion of the course, students will be able to

1. determine the determinant and inverse of a partitioned matrix.
2. construct Jordan canonical form and singular value decomposition of a matrix.
3. derive matrix inequalities using Schur complements, Hadamard and Kronecker products.
4. identify positive semidefinite matrices through eigenvalues, minors, and determinants and be able to compute Perron vector.
5. categorize matrix classes and use them in stability analysis and complementarity problems.

Course Code	MA746
Title of the Course and Type	Advanced Partial Differential Equations (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of the course is to
1. introduce the basic concepts on different types of partial differential equations.
2. explain different solutions techniques.
3. explore Sobolov spaces and their properties.
4. illustrate weak solutions and semigroup theory.
5. teach non-variational techniques to solve the partial differential equations.

Course Content:

Representation formulas for solutions: solutions of transport, Laplace, heat and wave equations – Complete integrals and characteristics odes of nonlinear first-order equations.

Hamilton-Jacobi-Bellmann equations – Conservation laws – Separation of variables – Similarity solution – Transform methods – Asymptotics and power series solution.

Introduction to Sobolov spaces: Weak derivatives – Approximations – Extensions – Traces and Sobolev inequalities.

Linear evolution equations of parabolic and hyperbolic types – Weak solutions – Hyperbolic system of 1st order equations – Semigroup theory.

Non-variational techniques: Banach fixed point theorem – Schauder's, Schaefer's fixed point theory – Subsolutions, supersolutions – Nonexistence of solutions – Viscosity solution – Riemann solution.

Reference Books:

1. L. C. Evans. *Partial Differential Equations*, 2nd edition, Graduate Studies in Mathematics, American Mathematical Society, 2010.
2. A. K. Nandakumaran and P. S. Datti. *Partial Differential Equations: Classical Theory with A Modern Touch*, 1st edition, Cambridge University Press, 2020.
3. K. S. Rao. *Introduction to Partial Differential Equations*, 3rd edition, PHI learning, 2010.
4. S. L. Sobolev. *Partial Differential Equations of Mathematical Physics*, New edition, Dover publication, 1990.
5. M. Bardi and I. Capuzzo-Dolcetta. *Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*, 2nd reprint edition, Springer, 2008.

Course Outcomes: On completion of the course, students will be able to
1. understand the basic concepts and classifications of partial differential equations.
2. illustrate the conservation laws and different solution methodologies.
3. explain Sobolov spaces and its various properties.
4. analyze weak solutions for different types problems.
5. use non-variational techniques for the solution methodologies.

Course Code	MA751
Title of the Course and Type	Non-linear Programming (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce the concepts of convex functions and to explore convex programming problems.
2. discuss the optimality conditions for the constrained and unconstrained nonlinear programming problems.
3. introduce Lagrange multipliers and Karush-Kuhn-Tucker conditions for constrained optimization problems.
4. study the algorithms to solve nonlinear programming problems.
5. analyze the theory of duality in nonlinear programming.

Course Content:

Problem statement – Local and global optimality – Convex sets – Convex functions and their properties – Differentiable convex functions – Convex programming problems.

Unconstrained optimization of functions of several variables – First and second order optimality conditions – Method of steepest descent – Newton's method – Conjugate gradient method.

Constrained nonlinear optimization problems – Equality and inequality constraints – Method of Lagrange multipliers – Karush-Kuhn-Tucker conditions.

Algorithms for constrained optimization – Frank-Wolfe's method – Projected gradient methods – Beale's method – Penalty methods.

Duality in nonlinear programming – Lagrangian dual problem – Duality theorems – Saddle point optimality conditions – Special cases – Linear and quadratic programming.

Reference Books:

1. M. S. Bazaraa, H. D. Sherali and C. M. Shetty. *Nonlinear Programming: Theory and Algorithms*, 3rd edition, Wiley, 2006.
2. E. K. P. Chong and S. H. Zak. *An Introduction to Optimization*, 3rd edition, Wiley, 2011.
3. D. G. Luenberger and Y. Ye. *Linear and Nonlinear Programming*, 5th edition, Springer, 2021.
4. S. S. Rao. *Engineering Optimization: Theory and Practice*, 5th edition, Wiley, 2021.
5. N. Andreasson, A. Evgrafov and M. Patriksson, *An Introduction to Continuous Optimization: Foundations and Fundamental Algorithms*, Illustrated edition, Dover Publication, 2020.

Course Outcomes: On completion of the course, students will be able to

1. determine convex sets and convex functions and to solve convex programming problems.
2. verify the optimality conditions of nonlinear programming problems and apply different methods to solve unconstrained optimization problems.
3. use the method of Lagrange multipliers and Karush-Kuhn-Tucker conditions to find an optimum solution of a nonlinear programming problems.
4. solve constrained nonlinear programming using Frank-Wolf's, projected gradient, Beale and penalty methods.
5. construct the dual of a non-linear programming problem and to apply duality results to solve them.

Course Code	MA752
Title of the Course and Type	Advanced Fuzzy Mathematics and its Applications (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce operations on fuzzy numbers such as addition, subtraction, multiplication, division, fuzzy max and fuzzy min.
2. explain different methods on ranking of fuzzy numbers using degree of optimality.
3. introduce different methods on ranking of fuzzy numbers using comparison function and scoring functions.
4. analyze different fuzzy decision-making methods using arithmetic operations.
5. discuss different fuzzy decision-making methods using max-min composition and converting fuzzy numbers to crisp scores.

Course Content:

Fuzzy arithmetic operations – Addition, subtraction, multiplication and division of fuzzy numbers, fuzzy max and fuzzy min – L-R and triangular (or trapezoidal) fuzzy numbers.

Ranking using degree of optimality – Watson et al., Baldwin and Guild's, Yager's, Kerre's, Adamo's, Buckley-Chana's and Mabuchi's approaches.

Ranking using comparison function – Ranking using left and right scores – Jain's approach, Chen's approach, Chen and Hwang's approach – Ranking with centroid index – Yager's centroid index and Murakami et al.'s approach.

Fuzzy simple additive weighting methods – Dubois and Prade's approach, Bonissone's approach – Analytic hierarchical process methods – Saaty's AHP approach, Laarhovan and Pedrycz's approach, Buckley's approach.

Maximin methods – Bellman and Zadeh's approach – Yager's approach – A new approach to fuzzy MADM problems – Converting linguistic terms to fuzzy numbers – Converting fuzzy numbers to crisp scores, the algorithm.

Reference Books:

1. S. J. Chen and C. L. Hwang, *Fuzzy Multiple Attributes Decision Making*, Springer, 1992.
2. G. J. Klir, Bo Yuan, *Fuzzy Sets and Fuzzy Logic - Theory and Applications*, Pearson Education, 2015.
3. T. J. Ross, *Fuzzy Sets and Fuzzy Logic with Engineering Applications*, Wiley, 2021.
4. H. J. Zimmermann, *Fuzzy Set Theory and its Applications*, Springer, 2014.
5. K. P. Yoon, C. L. Hwang, *Multiple Attribute Decision Making: An Introduction*, Sage publication, 1995.

Course Outcomes: On completion of the course, students will be able to

1. apply operations on fuzzy numbers such as addition, subtraction, multiplication, division, fuzzy max and fuzzy min.
2. identify different methods on ranking of fuzzy numbers using degree of optimality
3. compare different methods on ranking of fuzzy numbers using comparison function and scoring functions.
4. illustrate different fuzzy decision-making methods using arithmetic operations
5. classify different fuzzy decision-making methods using max-min composition and converting fuzzy numbers to crisp scores.

Course Code	MA753
Title of the Course and Type	Matrix Theory and Stochastic Programming (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. introduce probability theory.
2. discuss the random vectors and distributions.
3. teach matrix theory.
4. explore vector spaces and inner product spaces.
5. analyze the discrete moment problems using Linear programming.

Course Content:

Probability space – Events – Sample space.

Discrete and continuous random variables – Distributions – Moments.

Singular value decomposition – Canonical forms – Generalized inverse.

Vector spaces – Inner product spaces.

Primal and dual simplex methods – Integer programming – Simulation – Discrete moment problem.

Reference Books:

1. I. N. Herstein, *Topics in Algebra*, 2nd edition, Wiley, 2006.
2. R. Hogg, J. Mckean and A. Craig, *Introduction to Mathematical Statistics*, 8th edition, Pearson, 2018.
3. J. M. Ortega, *Matrix Theory,* Springer, 2013.
4. A. Prekopa, *Stochastic Programming*, 1st edition, Springer, 1995.
5. H. A. Taha, *Operations Research: An Introduction*, 10th edition, Pearson Education, 2019.

Course Outcomes: On completion of the course, students will be able to
1. understand probability theory in depth.
2. explain the random vectors and distributions.
3. utilize matrix theory for linear programming.
4. work with vector spaces and inner product spaces.
5. solve the discrete moment problems using linear programming.

Course Code	MA754
Title of the Course and Type	Advanced Numerical Analysis for Singularly Perturbed Differential Equations (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. present motivation for studying singular perturbation problems.
2. introduce the fitted operator method for singularly perturbed differential equations.
3. explore various numerical methods for solving boundary value problems with singular perturbation.
4. discuss about special numerical methods for singular perturbation problems.
5. introduce finite element method for solving singular perturbation problem.

Course Content:

Simple examples of singular perturbation problem – Uniformly distributed numerical method for problems with initial and boundary layers – Initial value problems – Some uniformly convergent difference schemes – Constant fitting factors – Optimal error estimates.

Boundary value problems – Constant fitting factors for a self-adjoint problem – Non-self-adjoint problem – Self-adjoint and non-self-adjoint problems in conservation form – Problems with mixed boundary conditions – Fitted verses standard method – Experimental determination of order of uniform convergence.

Fitted operator method – Fitted mesh method – Cubic spline method – Finite element method – Variable mesh method – Shooting method – Collocation method – Booster method – Boundary value technique – Initial value technique – Schwartz method – Convergence of the above methods – Reaction-diffusion, convection- diffusion, reaction- convection-diffusion type problems in one dimension.

Simple fitted mesh methods in one dimension – Convergence of fitted mesh finite difference methods for linear convection-diffusion problems in one-dimension – Linear convection-diffusion problems in two dimensions and their numerical solutions – Fitted numerical methods for problems with initial and parabolic boundary layers.

Finite element method and finite volume method for singularly perturbed ordinary and partial differential equations.

Reference Books:

1. J. J. H. Miller, E. O'Riordan and G. I. Shishikin, *Fitted Numerical Methods for Singular Perturbation Problems,* World Scientific, 2012.
2. C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, Springer, 1999.
3. L. C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, 1998.
4. R. C. Mc Owen, *Partial Differential Equations- Methods and Application*, Pearson Education, 2004.
5. E. P. Doolan, J. J. H. Miller and W. H. A. Schliders, *Uniform Numerical Methods for Problems with Initial and Boundary Layers*, Boole Press, 1980.

Course Outcomes: On completion of the course, students will be able to

1. design numerical methods with fitting factors for singularly perturbed initial value problems.
2. construct and analyze fitting factors for singularly perturbed boundary value problems.
3. solve various classes of singular perturbation problems numerically.
4. prove convergence of fitted mesh methods for one and two dimensional singular perturbation problems.
5. use finite element and finite volume methods for solving singular perturbation problems.

Course Code	MA755
Title of the Course and Type	Biorheology (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of the course is to

1. classify different types of fluids and flows.
2. explore the nature of blood and red cells.
3. study blood vessel walls and their theoretical models.
4. describe pulse waves and pulsatile flow.
5. discuss dilatation, thrombosis and permeability.

Course Content:

Fluid – Newtonian and Non-Newtonian flow – Laminar flow in a cylindrical tube – Turbulent flow – Suspensions – Hookean Solid – Viscoelasticity – Strain energy density function – Thixotrophy.

Blood – Aggregation and Sedimentation of red cells – Non-Newtonian viscosity – Thixotrophy and viscoelasticity – Factors affecting blood viscosity – Casson fluid – Plasma layer – Radial migration – Flow of red cell suspension in small tubes – Fahraeus effect, Fahraeus-Lindqvist effect – Wall surface effect – Copley-Scott Blair phenomenon – Disturbed flows of red cell suspensions – Viscosity of blood clots – Blood rheology at near-zero gravity.

Blood vessel walls – Forces in blood vessel walls – General theory of circumferential tension – Stress distribution in blood vessel walls – Incremental theory of blood vessel walls – Nonlinear theory of elastic deformation – Tethering effect on the stresses in blood vessels – Some rheological models of blood vessels.

Pulse – Theoretical studies of pulse waves – Oscillatory flow in a rigid tube – Wave propagation in elastic tubes – Pressure-flow relationship – Pulsatile flow in micro vessels.

Flow in a locally constricted tube – Post-stenotic dilatation – Flow at branching sites – Thrombosis – Atherosclerosis – Protein uptake by arterial wall – Permeability and pathways of macromolecules – Physical theory of vascular permeability to proteins – Stresses in the arterial wall as a cause of permeability.

Reference Books:

1. S. Oka, *Cardiovascular Hemorheology*, Cambridge University Press, 1981.
2. Y. C. Fung, *Biomechanics: Its foundations and Objectives*, Prentice Hall, 1973.
3. A. S. Logde, M. Renardy and J. A. Nohel, *Viscoelasticity and Rheology*, Academic Press, 1985.
4. D. Machin, *Biomathematics: An Introduction*, Springer, 1976.
5. J. C. Misra, *Biomathematics: Modeling and Simulation*, World Scientific, 2006.

Course Outcomes: On completion of the course, students will be able to

1. identify the types of fluids and flows.
2. figure out the blood components and the factors affecting blood.
3. choose research problem based on the size of blood vessel.
4. apply ideas of oscillatory flow and pulsatile flow in their research problems.
5. select research problems on permeability.

Course Code	MA756
Title of the Course and Type	Advanced Complex Analysis (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. introduce convex functions and prove Hadamard/Phragmen_Lindelof theorems.
2. study the space of continuous functions and prove Riemann mapping theorem.
3. discuss infinite products and characterise simply connected domains
4. teach various properties of harmonic functions
5. explore entire functions and prove Picard theorems.

Course Content:
Convex functions – Hadamard three lines and circles theorems – Phragmen-Lindelof theorem.

Space of continuous functions $C(G,\Omega)$ – Normal family – Spaces of analytic functions – Hurwitz's, Montel's and Riemann mapping theorems.

Infinite products – Weierstrass' factorization theorem – Factorization of the sine function – Runge's theorem, simply connected regions – Mittag-Leffler's theorem.

Harmonic functions – Maximum and minimum principles – Harmonic functions on a disk – Harnack's theorem – Dirichlet problem for disk – Green's function.

Entire functions – Jensen's formula – Bloch's, Picard and Schottky's theorems.

Reference Books:

1. J. B. Conway, *Functions of One Complex Variables – I*, 2nd edition, Narosa, 2002.
2. D. Ullrich, *Complex Made Simple*, Volume 97, American Mathematical Society, 2008.
3. H. L. Shin and E. Bernard, *Classical Complex Analysis*, Jones and Bartlett, 2011.
4. L. V. Ahlfors, *Complex Analysis*, 3rd edition, McGraw Hill Co, 2017.
5. W. Rudin, *Real and Complex Analysis*, 3rd edition, McGraw-Hill, 2017.

Course Outcomes: On completion of the course, students will be able to
1. prove properties of convex functions and prove Hadamard/Phragmen-Lindelof theorems.
2. explore various theorems about normal families and use them to prove Riemann mapping theorem.
3. obtain infinite products and characterize simply connected domains.
4. identify various properties of harmonic functions.
5. analyze various properties of entire functions and Picard theorems.

Course Code	MA757
Title of the Course and Type	Geometric Function Theory (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objectives of this course is to

1. discuss basic properties of univalent functions including those with special geometric properties.
2. introduce Lower's differential equations and to use them to investigate univalent functions.
3. explore Grunsky, Goluzin and Lebedev inequalities and other related inequalities.
4. illustrate subordination and majorization.
5. study the theory of differential subordination and to give some applications to univalent functions.

Course Content:

Univalent functions – Area, growth and distortion theorems – Coefficient estimates for univalent functions – Special classes of univalent functions (starlike, convex, close-to-convex, spiral, typically real functions).

Lowner's theory and its applications – Caratheodory convergence theorem – Density of slit mappings – Lowner's differential equation – Third coefficients – Radius of starlikeness – Rotation theorem – Robertson's conjecture – Outline of de Banges proof of Bieberbach conjecture.

Generalization of the area theorem – Faber polynomials and polynomial area theorem – Grunsky, Goluzin and Lebedev inequalities – Exponentiation of the Grunsky inequalities – Logarithmic coefficients.

Subordination – Coefficient inequalities and sharpened form of Schwarz lemma – Majorization – Univalent subordinate functions – Bernstein's theorem on integral means – Convolution and sections of univalent functions.

Theory of differential subordination – Jack-Miller-Mocanu lemma – Admissibility conditions – First and second order differential subordination and applications.

Reference Books:

1. P. Duren, *Univalent Functions*. Springer, 1983.
2. A. W. Goodman, *Univalent Functions I & II*. Mariner, 1983.
3. I. G. Ian and G. Kohr, *Geometric Function Theory in One and Higher Dimensions*. Marcel Dekker, 2003.
4. C. Pommerenke and G. Jensen, *Univalent Functions*. Van den Hoek and Ruprecht, 1975.
5. M. Rosenblum and J. Rovnyak, *Topics in Hardy Classes and Univalent Functions*. Birkhauser, 1994.

Course Outcomes: On completion of the course, students will be able to

1. prove basic properties of univalent functions including those with special geometric properties.
2. derive Lower's differential equations and to use them to investigate univalent functions.
3. analyze Grunsky, Goluzin, Lebedev inequalities and other related inequalities.
4. define subordination and majorization and use them to prove coefficient inequalities.
5. use the theory of differential subordination to prove properties of univalent functions.

Course Code	MA758
Title of the Course and Type	Approximation Theory (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. present the approximation of continuous functions by polynomials.
2. explain Chebyshev's alternation theorem and Haar's unicity theorem.
3. learn the best approximation by subsets of normed linear spaces and inner product spaces.
4. understand duality formula and proximinality in Banach spaces.
5. introduce upper, lower semi continuity of the metric projection maps and the continuous selections of the set valued maps.

Course Content:

Approximation by algebraic polynomials – Approximation of periodic functions – Convergence of Bernstein polynomials – Korovkin's theorem – Stone-Weierstrauss theorem.

Chebyshev's alternation theorem – General linear families – Haar system and its characterizations – Uniqueness of polynomials of best approximation – Strong unicity theorem – Harr's unicity theorem – Orthogonal polynomials – Legendre and Chebyshev polynomials.

Strictly convex and uniformly convex Banach spaces – Best approximation in inner product spaces – Best approximation from closed, convex subsets – Continuity of metric projection.

Best approximation by subspaces of Banach spaces – Duality formula – Proximinal sets – Chebyshev sets – Proximinality of weak* closed subspaces.

Upper semi continuity and lower semi continuity of the set-valued maps – Continuous selections and Lipschitz continuity of metric projections.

Reference Books:

1. W. Cheney and W. Light, *A Course in Approximation theory*, *Graduate Studies Mathematics,* American Mathematical Society, 2013.
2. F. Deutsch, *Best Approximation in Inner Product Spaces*, Springer, 2001.
3. S. Lang, *Real and Functional Analysis*, Springer, 1993.
4. I. Singer, *Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces*, Springer, 1970.
5. H. N. Mhaskar and D. V. Pai, *Fundamentals of Approximation Theory*, Narosa Publishing House, 2007.

Course Outcomes: On completion of the course, students will be able to
1. explain the approximation of continuous functions by polynomials.
2. find some applications of Chebyshev alternation theorem.
3. comprehend the best approximation from closed convex subsets of inner product spaces and continuity of metric projection.
4. understand the best approximation by subspaces of Banach spaces.
5. explain the semi continuity properties of set-valued maps and continuous selection of metric projection maps.

Course Code	MA759
Title of the Course and Type	Convex Analysis (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce Caratheodory's theorem and Helley's theorem.
2. present the convex functions and extremal structure of convex sets.
3. explore the extreme points and Krein Milman theorem.
4. understand the differentiability of convex functions.
5. explore the derivatives of norm functions.

Course Content:

Convex sets – Convex hull and affine hull – Convex subsets of $\mathbb{R}^n$ – Caratheodory's theorem – Helley's theorem – Convex hull of a compact set is compact.

Convex functions on $\mathbb{R}^n$ – One sided derivatives – Continuity and differentiability – Directional derivatives and subgradients – Differentiable convex functions – Inequalities.

Extremal subsets and extreme points of convex sets – Extreme points of unit ball of Banach spaces – Krein Milman theorem – Milman converse.

Differentiability of convex functions defined on Banach spaces – Sublinear functionals and one-sided derivatives – Gateaux and Frechet derivatives – Mazur's theorem.

Derivatives of the norm function and the duality maps – Smulian's characterization of the differentiability of the norm function.

Reference Books:

1. J. V. Tiel, *Convex Analysis*, John Wiley, 1984.
2. R. R. Phelps, *Convex Functions, Monotone Operators and Differentiability*, Springer Lecture Notes, 1993.
3. M. Fabian, P. Habala, P. Hajek, V. M. Santalucia, J.Pelant and V. Zizler: *Functional Analysis and Infinite-Dimensional Geometry*, CMS Books in Mathematics, 2001.
4. R. T. Rockafeller, *Convex Analysis*, Princeton University Press, 1972.
5. R. E. Megginson, *An Introduction to Banach Space Theory*, Springer, 2012.

Course Outcomes: On completion of the course, students will be able to

1. explain the convex sets, Caratheodory's theorem and Helley's theorem.
2. understand the directional derivatives and sub gradients of convex functions.
3. comprehend the extreme points of convex sets and know some applications of Krein-Milman theorem.
4. find Gateaux and Fréchet derivatives of convex functions defined on Banach spaces.
5. explain the derivatives of norm functions and duality maps.

Course Code	MA760
Title of the Course and Type	Graphs and Matrices (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. study block matrices, spectral theorem, eigenvalue inequalities and Perron-Frobenius Theorem.
2. understand graph theoretic properties through its associated graph matrices.
3. analyze the concepts of the Incidence matrix and the Adjacency matrix of a graph
4. introduce the notion of the Laplacian matrix of a graph and to discuss its properties.
5. investigate the Distance matrix of a graph and to study its determinant, inertia, and inverse.

Course Content:

Determinant and inverse of partitioned matrices – Schur complement of a block matrix – Spectral theorem – Rayleigh quotient for Hermitian matrix – Cauchy's Interlace theorem for eigenvalues – Perron-Frobenius theorem.

Basic notions in Graph theory – Euler's theorem – Line graphs – Operations on graphs – Complement, union and join of graphs – Graph products – Cartesian product and composition of graphs – Vertex/Edge independent sets.

Incidence matrix – Properties of incidence matrix – Adjacency matrix – Adjacency spectrum – Sachs' theorem – Determinant of the adjacency matrix of simple graphs.

Laplacian matrix – Properties of Laplacian matrix – Matrix-tree theorem – Laplacian eigenvalues of trees.

Distance matrix – Determinant and inverse of distance matrix of trees – Relation between distance and incidence matrix of trees – Inertia of distance matrix of a tree.

Reference Books:

1. R. B. Bapat, *Graphs and Matrices*, 2nd edition, Universitext, Springer,2014.
2. A. E. Brouwer and W. H. Haemers, *Spectra of Graphs*, Universitext, Springer, 2014.
3. D. Cvetkovic, P. Rowlinson and S. Simic, *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, 2010.
4. F. Harary, *Graph Theory*, Narosa Publishing House, 2001.
5. R. A. Horn and C. R. Johnson, *Matrix Analysis*, 2nd edition, Cambridge University Press, 2012.

Course Outcomes: On completion of the course, students will be able to

1. compute determinant and inverse of a block matrix using Schur complement
2. apply Perron-Frobenius theorem and the Rayleigh quotient to several graph matrices to obtain bounds for graph theoretic parameters
3. give the rank and the column space of the Incidence matrix of a graph and derive eigenvalue inequalities between matrices of graphs and its induced subgraphs using interlacing theorem
4. find coefficients of the characteristic polynomial of the adjacency matrix and relate cofactors of the Laplacian matrix of a graph with its spanning trees
5. determine inertia, determinant, and inverse formula for the distance matrix of a tree.

Course Code	MA761
Title of the Course and Type	Fitted Mesh and Fitted Operator Methods for Singular Perturbation Problems (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce singularly perturbed differential equations.
2. discuss fitted mesh methods for reaction-diffusion problems.
3. explore fitted mesh methods for convection-diffusion problems.
4. introduce fitted operator methods for solving singularly perturbed problems.
5. analyse necessary conditions for stability of fitted operator methods.

Course Content:

Motivation for the study of singular perturbation problems – simple examples of singular perturbation problems – Numerical methods for singular perturbation problems – Simple fitted mesh methods in one dimension.

Convergence of fitted mesh finite difference methods for linear reaction – diffusion problems in one dimension.

Convergence of fitted mesh finite difference methods for linear convection – diffusion problems in one dimension.

Initial value problems – Some uniformly convergent difference schemes – Constant fitting factors – Optimal error estimates.

Boundary value problems – Introduction - Examples – Asymptotic Expansions – Historical Perspective – Necessary conditions– Constant fitting factor.

Reference Books:

1. J. J. H. Miller, E. O' Riordan and G. I. Shishkin, *Fitted Numerical Methods for Singular Perturbation Problems – Error estimates in the Maximum Norm for Linear Problems in One & Two Dimensions*, World Scientific Publishers, 2012.
2. E. P. Doolan, J. J. H. Miller and W. H. A. Schliders, *Uniform Numerical Methods for Problems with Initial and Boundary Layers,* Boole Press, 1980.
3. R. E. O'Malley, *Introduction to Singular Perturbations,* Academic Press, 1974.
4. R.E. O'Malley, *Singular Perturbation Methods for Ordinary Differential Equations*, 1st edition, Springer, 1991.
5. P. A. Farrel, A. F. Hegarty, J. J. H. Miller, E. O' Riordan and G. I. Shishkin, *Robust Computational Techniques for Boundary Layers,* Chaman & Hall/CRC, 2000.

Course Outcomes: On completion of the course, students will be able to

1. determine solution behaviour of singular perturbation problems.
2. design fitted mesh methods for reaction-diffusion equations.
3. construct fitted mesh methods for convection-diffusion equations.
4. derive error estimates of numerical schemes with constant fitting factors.
5. solve boundary value problems with constant fitting factors.

Course Code	MA762
Title of the Course and Type	Queueing Theory (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. acquire knowledge of basic queueing models.
2. impart understanding on busy period analysis.
3. introduce the queueing models with general service time distributions.
4. discuss the phase-type distribution and batch Markovian arrival process.
5. familiarize matrix analytic method for various queueing models.

Course Content:

Arrival and departure processes – Single and multiple channel queues – Queues with finite waiting room – Little's formula.

Waiting time distributions – Busy period analysis – Erlang's loss formula (transient solutions for M/M/1 model) – Self-serving queues.

M/G/1 queueing model – Pollaczek-Khintchine formula – Steady-state system size probabilities – Waiting time distributions – Generalization of Little's formula – Busy period analysis.

Continuous phase-type (CPH) distribution – CPH renewal process – Continuous-time batch Markovian arrival process (BMAP) – Counting processes associated with BMAP – Generation of MAP processes for numerical purposes.

Matrix-analytic methods (continuous-time): M/G/1-type (scalar case, matrix case) – GI/M/1-type (scalar case, matrix case) – QBD process (scalar case, matrix case) – Busy period in QBD-type queues.

Reference Books:

1. J. F. Shortle, J. M. Thompson, D. Gross and C. M. Harris, *Fundamentals of Queueing Theory*, 5th edition, John Wiley and Sons, 2017.
2. J. Medhi, *Stochastic Models in Queueing Theory*, 2nd edition, Academic Press, 2002.
3. Qi-Ming He, *Fundamentals of Matrix - Analytic Methods*, 1st edition, Springer, 2014.
4. S. R. Chakravarthy, *Introduction to Matrix-Analytic Methods in Queues 1, Analytical and Simulation Approach – Basics*, 1st edition, John Wiley & Sons, 2022.
5. S. R. Chakravarthy, *Introduction to Matrix-Analytic Methods in Queues 2, Analytical and Simulation Approach – Queues and Simulation*, 1st edition, John Wiley & Sons, 2022.

Course Outcomes: On completion of the course, students will be able to

1. analyze the birth and death processes and their applications.
2. derive various performance measures of the queueing model with general service times.
3. explain continuous phase-type distribution and BMAP.
4. illustrate matrix analytic method for various queueing models
5. construct queueing systems from real-life.

Course Code	MA763
Title of the Course and Type	Finite Element Methods (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. discuss the variational formulation and finite element method (FEM) for the model problem with piecewise linear functions.
2. introduce error estimates for FEM solutions.
3. teach semi-discretization in space and time discretization FEM.
4. discuss stabilization techniques such as the streamline diffusion method to solve hyperbolic problems.
5. introduce discontinuous Galerkin (DG) methods for one-dimensional model problems.

Course Content:

Introduction to FEM for elliptic problems – Variational formulation of a one-dimensional model problem – FEM with piecewise linear functions – Error estimate for FEM – FEM for Poisson equation – Neumann problem – Remarks on programming.

Abstract formulation of FEM for elliptic problems – Existence and uniqueness of finite element solution using Lax - Milgram lemma – Discretization and error estimate – Energy norm – Regularity requirements – Some examples of finite element spaces.

One-dimensional model problem – FEM for parabolic problems – Semidiscretisation in space and fully discretisation method – Backward Euler and Crank-Nicolson methods.

Introduction – FEM for hyperbolic problems –Convection-diffusion problem – General remark on numerical method for hyperbolic equations – Standard Galerkin method – Streamline diffusion method.

One-dimensional model problem – Discontinuous Galerkin method for the one-dimensional model problem –Class of DG methods – Computing the matrix A and right-hand side vector b – Convergence of the DG method – Remarks on programming.

Reference Books:

1. C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Dover Publications, 2009.
2. J. N. Reddy, *An Introduction to Finite Element Method*, McGraw Hill, 1993.
3. S. C. Brenner and L. R. Scott, *The Mathematical Theory of Finite Element Methods*, Springer, 2002.
4. Z. Chen, *Finite Element Methods and Their Applications*, Springer, 2005.
5. B. Riviere, *Discontinuous Galerkin Method for Solving Elliptic and Parabolic Equations*, SIAM, 2008.

Course Outcomes: On completion of the course, students will be able to

1. demonstrate the ability to solve elliptic problems using the finite element method.
2. analyze and estimate errors in FEM solutions.
3. apply semi-discretization and fully discretization methods effectively to solve parabolic problems.
4. implement and use stabilization techniques for accurately solving hyperbolic problems with FEM.
5. apply DG methods for one-dimensional model problems and evaluate their convergence.

Course Code	MA764
Title of the Course and Type	Particulate Processes: Theory and Modelling (Program Elective)
Prerequisites	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce particulate processes and population balance problem.
2. explore different particulate dynamical models.
3. teach existence and uniqueness of regular solutions.
4. train numerical approximations using sectional methods and methods of moment.
5. discuss stability, consistence, and convergence of solutions.

Course Content:

Introduction to particulate processes – Different mathematical models representing particulate processes both discrete and continuous – Aggregation, fragmentation, collision-induced process.

Discrete particulate process – Condensing coagulation – Oort-Hulst-Safronov model – Redner-Ben-Abraham-Kahng cluster system – Existence and uniqueness of solutions.

Continuous particulate processes – Existence, uniqueness – Equilibria with and without balanced condition – Equilibria via Lyapunov functional – Occurrence finite-time and instantaneous gelling – Shattering – Self-similar solutions and asymptomatic behaviors.

Numerical discretization – Sectional methods – Method of moments – Semi-analytical methods – Monte-Carlo approach.

Relation between the discrete and continuous models – Passing the limits – Stability – Convergence.

Reference Books:

1. D. Ramakrishna, *Population balances: Theory and Applications to Particulate Systems in Engineering*, Academic Press, 2000.
2. J. Banasiak, W. Lamb and P. Laurencot, *Analytic Methods for Coagulation-Fragmentation Models – I*, CRC Press, Chapman & Hall, 2019.
3. J. Banasiak, W. Lamb and P. Laurencot, *Analytic Methods for Coagulation-Fragmentation Models – II,* CRC Press, Chapman & Hall, 2019.
4. P. B. Dubovskii, *Mathematical Theory of Coagulation*, Seoul National University press, 1994.
5. H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2011.

Course Outcomes: On completion of the course, student will be able to

1. understand and model different particulate events arising in industry.
2. Explore various population balance models and their relation with real-life.
3. analyze different aspects of solutions (existence, uniqueness, steady-state, self-similarity, non-existence).
4. design efficient and accurate numerical methods for solving coupled problems.
5. prove stability of the numerical scheme and convergence of the solutions.

Course Code	MA765
Title of the Course and Type	Introduction to Hydrodynamic Stability (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. introduce the concept of stability of fluid flows.
2. study the basics of linear stability theory.
3. examine several instabilities such as, Rayleigh-Taylor and Kelvin-Helmholtz instability.
4. derive Rayleigh and Orr-Sommerfeld equations.
5. discuss numerical techniques for computationally determining stability.

Course Content:
Basic review of fluid dynamics – Navier-Stokes equations for Newtonian and non-Newtonian fluids – concept of instability.

Bifurcations – Phase space and phase portrait – Stability and instability – Linearized problems and normal mode analysis.

Kelvin-Helmholtz and Rayleigh-Taylor instabilities – Rayleigh-Benard convection (thermal instability) – Taylor-Couette centrifugal instability.

Definition of stability – Disturbance equation – Critical Reynolds numbers – Development of instabilities in space and time.

Stability of parallel shear flows – Inviscid theory and viscous theory – Spatio-temporal stability analysis – Absolute and convective instabilities – Computational methods for hydrodynamic stability.

Reference Books:

1. P. Drazin, *Introduction to Hydrodynamic Stability*, Cambridge University Press, 2002.
2. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Dover Publications, 2013.
3. F. Charru, *Hydrodynamic Instabilities*, Cambridge University Press, 2011.
4. P. J. Schmidt and D. S. Henningson, *Stability and Transition in Shear Flows*, Springer, 2012.
5. W. O. Criminale, T.L. Jackson and R. D. Joslin, *Theory and Computation in Hydrodynamic Stability*, Cambridge University Press, 2003.

Course Outcomes: On completion of the course, students will be able to
1. develop an understanding by which “small" perturbations grow, saturate and modify fluid flows.
2. apply linear stability to determine the onset of instability.
3. determine the nature of instabilities.
4. recognize importance of unstable nonlinear solutions.
5. build a basic foundation for addressing/approaching research problems on laminar to turbulent transition.

Course Code	MA766
Title of the Course and Type	Fixed Point Theory and its Applications (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. develop an understanding of Banach contraction principle and its extensions.
2. study Caristi's fixed point theorem and Ekeland principle.
3. examine Ky Fan's best approximation theorems in Hilbert spaces.
4. understand the KKM-Map principle, its extensions and applications.
5. study fixed point theorems in partially ordered spaces and other abstract spaces.

Course Content:

Banach contraction principle and its extension – Applications to system of linear equations – Differential equations – Integral equations.

Caristi's fixed point theorem – Ekeland principle – Fixed points of non-expansive and set valued maps – Brouwer-Schauder fixed point theorems.

Ky Fan's best approximation theorems in Hilbert spaces – Applications to fixed point theorems – Prolla's theorem and extensions.

The KKM- Map principle – Extensions of the KKM-Map Principle, its applications, variants and applications.

Fixed point theorems in partially ordered spaces and other abstract spaces – Application of fixed-point theory to game theory.

Reference Books:

1. K. C. Border, *Fixed Point Theorems with Applications to Economics and Game Theory*, Cambridge University Press, 1989.
2. K. Goebel and W. A. Kirk, *Topics in Metric Fixed-Point Theory*, Cambridge Studies in Advanced Mathematics, Vol. 28, Cambridge University Press, 1990.
3. M. A. Khamsi and W. A. Kirk, *An Introduction to Metric Spaces and Fixed-Point Theory*, Wiley, 2001.
4. W. A. Kirk and B. Sims, *Handbook of Metric Fixed-Point Theory*, Springer, 2001.
5. S. Singh, B. Watson and P. Srivastava, *Fixed Point Theory and Best Approximation: The KKM-Map Principle*, Kluwer Academic Publishers, 1997.

Course Outcomes: On completion of the course, students will be able to
1. apply the principle to solve systems of linear equations, differential equations and integral equations.
2. find fixed points of non-expansive and set-valued maps.
3. apply Ky Fan's theorems to fixed point problems and understand Prolla's theorem.
4. use the variants of the KKM-Map principle to solve practical problems.
5. analyze fixed point theorems in partially ordered spaces and other abstract spaces.

Course Code	MA767
Title of the Course and Type	Advanced Functional Analysis (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. introduce topological vector spaces, their separation properties and linear mappings.
2. study the Hahn-Banach separation and related theorems.
3. delve into Schauder bases, Mazur's basic theorem, block basis sequences and relevant inequalities.
4. study linear operators, adjoints, compact and finite rank operators on Banach spaces.
5. teach the fundamental aspects of spectral theory for operators on Banach spaces.

Course Content:

Topological vector spaces – Separation properties – Linear mappings – Finite dimensional spaces – Metrizability, boundedness and continuity – Locally convex spaces, convex sets and semi norms – Normed and normable spaces – Projective and inductive topologies – Separation of convex sets – Quotient spaces.

Hahn-Banach separation theorem – Weak and weak* topologies in normed linear spaces – Reflexive spaces – Banach-Alaoglu theorem – Goldstine's theorem – Krein-Milman´s theorem – Equivalence of reflexivity and weak compactness of the closed unit ball – Schur property of l_1.

Schauder bases – Shrinking and boundedly complete bases – Mazur's basic sequence theorem – Block basis sequence – Pitt's theorem – Khintchine's inequality – Schauder's basis for $C[0,1]$ – Subspaces of l_p and c_0 – Complemented subspaces of l_p and c_0 – Special properties of c_0 , l_1, l_∞ and L_p spaces – Basic inequalities.

Linear operators on Banach space – Adjoint of a linear operator – Compact and finite rank operators – Fredholm operators – Invariant subspaces – Weakly compact operators.

Elementary properties of spectral theory for operators on Banach space – Ideals and quotients – Spectrum – Spectrum of a linear operator – Spectral theory of a compact self-adjoint and compact normal operators.

Reference Books:

1. N. L. Carothers, *A Short Course on Banach Space Theory*, London mathematical Society student texts, Cambridge University Press, 2005.
2. J. B. Conway, *A Course in Functional Analysis*, 2nd edition, Graduate Texts in Mathematics, Springer, 1990.
3. M. Fabian, P. Habala, P. Hajek, V. M. Santalucia, J. Pelant and V. Zizler, *Functional Analysis and Infinite Dimensional Geometry*, CMS Books in Mathematics, Springer, 2001.
4. R. E. Megginson, *An Introduction to Banach Space Theory*, Graduate Texts in Mathematics, Springer, 1998.
5. W. Rudin, *Functional Analysis*, 2nd edition, Mc-Graw Hill, 1991.

Course Outcomes: On completion of the course, students will be able to

1. describe locally convex spaces and identify convex sets and semi-norms.
2. apply the Hahn-Banach separation theorem, Banach-Alaoglu, Goldstine's, and Krein-Milman's theorem to solve problems in normed linear spaces.
3. utilize Mazur's basic sequence theorem and block basis sequences in problem-solving.
4. analyze linear operators on Banach spaces, including their adjoints.
5. apply the principles of spectral theory for operators on Banach spaces.

Course Code	MA768
Title of the Course and Type	Theory and Geometry of Banach Spaces
Prerequisite	Nil
Credits (L-T-P)	3 – 0 – 0

Course Learning Objectives: The objective of this course is to

1. study weak and weak* topologies, Banach-Alaoglu, Goldstine's and their implications.
2. discuss extreme points, Krein-Milman's, Eberlein-Smulian and Bishop-Phelps theorem.
3. explore various notions of rotundity and modulus of rotundity.
4. explain various smoothness notions, their duality relations with rotundity and smooth renormings.
5. introduce M-ideals, M-summands, and L-summands in Banach spaces and their basic properties.

Course Content:

Dual spaces – Reflexivity – Quotient spaces – Locally convex spaces – Weak and weak* topologies – Banach-Alaoglu theorem – Goldstine's theorem.

Extreme points – Krein-Milman´s theorem – Eberlein-Smulian theorem – Equivalence of reflexivity and weak compactness of the closed unit ball – Bishop-Phelps theorem.

Rotundity – Modulus of rotundity – Uniform rotundity – Milman-Pettis theorem – Locally uniform rotundity – Weakly uniform rotundity – Basic rotund renormings.

Smoothness – Duality relation of rotundity and smoothness – Modulus of smoothness – Uniform smoothness – Spherical image map – Frechet smooth – Basic smooth renormings.

Projections and complementability – Ball intersection properties – M-ideals – M-summands and L-summands in Banach space and its basic properties.

Reference Books:

1. R. Deville, G. Godefroy and V. Zizler, *Smoothness and Renormings in Banach spaces*, Pitman monographs and surveys in pure and applied mathematics, Vol. 64, Longman Scientific and Technical, 1993.
2. J. Diestel, *Geometry of Banach spaces – Selected Topics*, Lecture Notes in Mathematics, Springer, 1975.
3. P. Harmand, D. Werner and W. Werner, *M-ideals in Banach Spaces and Banach Algebras,* Lecture Notes in Mathematics, Springer, 1993.
4. W. B. Johnson and J. Lindenstrauss, *Basic Concepts in the Geometry of Banach spaces*, Handbook of the Geometry of Banach spaces, Vol. 1, North-Holland, 2001.
5. R. E. Megginson, *An Introduction to Banach Space Theory, Graduate Texts in Mathematics*, Vol. 183, Springer, 1998.

Course Outcomes: On completion of the course, students will be able to

1. implement Banach-Alaoglu and Goldstine's theorem to solve problems involving dual spaces and weak topologies.
2. apply Krein-Milman's, Eberlein-Smulian and Bishop-Phelps theorem to problems involving extreme points and convex sets.
3. explain and apply the concepts of rotundity, modulus of rotundity and uniform rotundity.
4. understand smoothness concepts, including modulus of smoothness and spherical image maps.
5. use ball intersection properties, M-ideals, M-summands, and L-summands to solve relevant problems.

Course Code	MA769
Title of the Course	Applied Functional Analysis (Program Elective)
Prerequisite	Nil
Credits (L-T-P)	3 – 0- 0

Course Learning Objectives: The objective of this course is to
1. discuss the test functions and various distributions.
2. analyze different properties of functions and related theorems.
3. familiarize with different spaces and properties.
4. study solutions of differential equations in Sobolov spaces.
5. explore higher dimensional real-life problems

Course Content

Test functions and distributions – Convolution of distributions – Tempered distributions.

Definition and basic properties – Approximations by smooth functions – Extension theorems,

Weak topologies – L^p spaces – Hilbert spaces –Compact operators – Hille-Yosida theorem.

Examples of elliptic boundary value problems – Existence and regularity of weak solutions – Maximum principle – Sobolev spaces and the variational formulation of boundary value problems in one dimension.

Variational formulation of elliptic boundary value problems in N −dimensions – Applications to real life problems – Heat equation – Wave equation.

Reference Books:

1. S. Kesavan, *Topics in Functional Analysis and Applications*, New Age International Publishers, 2015.
2. H. Brezis, *Functional Analysis, Sobolev Space and Partial Differential Equations*, Springer, 2011.
3. M. Renardy and R. C. Roger, *An Introduction to Partial Differential Equations*, Springer, 2004.
4. G. Leoni, *A First Course in Sobolev Spaces*, Graduate Studies in Mathematics, Springer, 2009.
5. S. Salsa, *Partial Differential Equation in Action: From Modelling to Theory*, Springer, 2008.

Course Outcomes: On completion of the course, students will be able to
1. identify and analyze test functions and related distributions.
2. analyze various functions and theorems.
3. explain different function spaces.
4. analyze the solutions of boundary value problems in one dimensions.
5. use the solution strategies to solve problems in higher dimensions.

Course Code	MA781
Title of the Course and Type	Research Methodology in Mathematical Sciences
Prerequisite	Nil
Credits (L-T-P)	4 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. teach the research ethics, plagiarism rules and preparation of manuscripts.
2. discuss different concepts on technical writing.
3. explore scientific writing skills and policies for dissemination of results.
4. familiarize with simulation tools for mathematical computations.
5. equip to use documentation software for typesetting manuscripts and slides.

Course Content:
Research methodology and ethics – Nature of research qualifications – Review of literature – Basic research – Applied research – Research methods – Research process – Research ethics – Intellectual properties.

English for technical writing – Research writing – Writing process – Organizing an argument – Academic integrity.

Scientific writing and presentation – Identifying journals – Abstracting and indexing of journals – Plagiarism rules – Organizing the document – Writing and publishing research papers related to Ph.D. thesis.

Simulation with software – Numerical matrix computation – Data fitting techniques and practical case studies – Root finding – Optimization and practical case studies – Solving ordinary and partial differential equations.

Documentation – Introduction to LaTeX – Changing the type style – Symbols – Mathematical formulae – Commands and environments – Figures and floating bodies – Line and page breaking – Fonts, errors and the bibliography database – Creating slides using Beamer.

Reference Books:

1. D. H. Mcburney, *Research Methods*, Thomson Asia Pvt Ltd, 2002.
2. J. A. Tarutz, *Technical Editing: The Practical Guide for Editors and Writers*, Hewlett-Packard Press, 1992.
3. R. A. Day, N. Sakaduski, *Scientific English: A Guide for Scientists and Other Professionals*, 3rd edition, Greenwood Press, 2011.
4. A. Gilat, *MATLAB – An introduction with applications*, 6th edition, Wiley, 2023.
5. L. Lamport, LATEX: A Document Preparation System. Addison-Wesley, 1994.

Course Outcomes: On completion of the course, students will be able to
1. understand research ethics, plagiarism and intellectual property rights.
2. analyse multi-variable functions, metric spaces, holomorphic functions, groups, rings and fields.
3. solve multiple integrals and higher order/simultaneous differential equations.
4. simulate mathematical problems using software.
5. prepare manuscripts and slides using LaTeX.

Course Code	MA782
Title of the Course	Advanced Mathematics
Prerequisite	Nil
Credits (L-T-P)	4 – 0 – 0

Course Learning Objectives: The objective of this course is to
1. explore functions of several variables and their applications in different fields.
2. discuss metric spaces and key theorems in real analysis and complex analysis.
3. disseminate knowledge on algebraic structures such as groups, rings and fields.
4. deliver the concepts of differential equations and their numerical solutions.
5. discuss probability theory and statistical inference to analyse random phenomena and data.

Course Content:

Functions of several variables, linear transformations, differentiation, the inverse function theorem, the implicit function theorem – Differentiation of integrals – Vector valued functions and their derivatives, line and surface integrals – Green's theorem, Gauss divergence theorem, Stokes' theorem and applications.

Metric spaces, completeness, compactness, connectedness, Banach contraction principle, its extension and applications – Arzela-Ascoli theorem – Baire category theorem – Complex integration, Liouville's theorem, maximum modulus principle, entire functions and meromorphic functions – Laurent series and singularities, Mittag-Leffler theorem, conformal mappings, examples, and properties.

Invariant subspaces – Direct-sum decompositions – Invariant direct sums – Primary decomposition theorem – The Jordan canonical form – Group theory - Sylow's theorem – Fundamental theorem of finite abelian groups – Ring theory – Ideals and factor rings – Prime and maximal ideals– Domains - Field extensions.

Method of solutions of successive approximations and Picard's theorem – Numerical linear algebra – Interpolation and approximation – Numerical differentiation and integration – Solutions of nonlinear equations, ordinary and partial differential equations – Error analysis and stability.

Random variables probability distributions – Law of large numbers and central limit theorem – Discrete and continuous distributions – Transformations and functions of random variables – Statistical inference – Confidence intervals and testing of hypothesis.

Reference Books:

1. C. H. Edwards, *Advanced Calculus of Several Variables*, Dover Publications, 1995.
2. H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, 4th edition, Pearson, 2015.
3. J. B. Conway, *Functions of One Complex Variable*, 2nd edition, Graduate Texts in Mathematics, Springer, 2012.
4. S. Axler, *Linear Algebra Done Right*, 4th edition, Undergraduate Texts in Mathematics, Springer, 2024.
5. D. S. Dummit and R. M. Foote. *Abstract Algebra*, 3rd edition, Wiley, 2011.
6. R. L. Burden, J. D. Faires and A.M. Burden, *Numerical Analysis*, 10th edition, Cengage Learning, 2015.
7. M. Tenenbaum and H. Pollard, *Ordinary Differential Equations*, Dover Publications, 1985.
8. A. M. Mood, F. A. Graybill and D. C. Bose, *Introduction to the Theory of Statistics*, McGraw-Hill, 1974.

Course Outcomes: On completion of this course, students will be able to
1. solve problems involving functions of several variables and related theorems.
2. apply key theorems of metric spaces and complex analysis to solve mathematical problems.
3. utilize and understand algebraic structures in group, ring and field theory.
4. solve differential equations numerically with error analysis and stability.
5. apply principles of probability and statistics to random variables, and testing of hypothesis.