MASTER OF SCIENCE IN MATHEMATICS

(M.Sc., Mathematics)

Syllabus for

Credit Based Flexible Curriculum

(From the academic year 2022 – 23 onwards)

BOARD OF STUDIES

DEPARTMENT OF MATHEMATICS

NATIONAL INSTITUTE OF TECHNOLOGY TIRUCHIRAPPALLI,

TIRUCHIRAPPALLI – 620015, TAMIL NADU, INDIA

THE INSTITUTE

VISION, MISSION, CORE VALUES AND GOALS

NIT Tiruchirappalli, through its Vision, Mission and Core Values, defines herself as:

- An Indian institution with world standards
- A global pool of talented students, committed faculty and conscientious researchers
- Responsive to real – world problems and, through a synergy of education and research, engineer a better society

VISION

- To be a university globally trusted for technical excellence where learning and research integrate to sustain society and industry.

MISSION

- To offer undergraduate, postgraduate, doctoral, and modular programmes in multi – disciplinary / inter – disciplinary and emerging areas.
- To create a converging learning environment to serve a dynamically evolving society.
- To promote innovation for sustainable solutions by forging global collaborations with academia and industry in cutting – edge research.
- To be an intellectual ecosystem where human capabilities can develop holistically.

CORE VALUES

Integrity: Honest in intention, fair in evaluation, transparent in deeds and ethical in our personal and professional conduct that stands personal and public scrutiny.

Excellence: Commitment to continuous improvement coupled with a passion for innovation that drives the pursuit of the best practices; while achievement is always acknowledged, merit will always be recognized.

Unity: Building capacity through trust in others' abilities and cultivating respect as the cornerstone of collective effort.

Inclusivity: No one left behind; no one neglected; none forgotten in the mission of nation – building through higher learning.

GOALS

International Accreditation and Ranking in tertiary education largely guide goal setting. The perception built by the stakeholders, crucially influence the process of repositioning. Benchmarking with global universities who are in the top 200 in world rankings in terms of teaching, innovation and research, funding, and internationalization. Hence, the need to set the following goals:

- Attracting top talent and global collaborations
- Building world – class research infrastructure to facilitate multi - / inter - / trans - disciplinary research
- Initiatives towards financial sustainability
- Social outreach activities of national / international importance
- Top 10 in India ranking in Engineering Discipline
- Top 500 in World Ranking in five years

THE DEPARTMENT OF MATHEMATICS

The Department of Mathematics is one of the pioneering and the most distinguished departments in National Institute of Technology, Tiruchirappalli. Applying a multi – disciplinary research and teaching methods, the department strongly believes in finding mathematical solutions for various social – economic, technological, and work – related processes and challenges. With fourteen faculty members representing major areas of mathematics, the department is at the forefront of cutting – edge research as well as teaching and innovation.

The department is committed to outstanding graduate training to produce leading scholars in various fields of mathematics. Since its inception, the department molds Ph. D graduates to carry out challenging research problems which have wide ranging industrial and social implications. Students are provided with ample opportunities to improve their research, teach courses, and participate in conferences/seminars.

VISION

- To be a global Centre of Excellence in mathematics and scientific computing for the growth of science and technology.

MISSION

- Committed to the cause of quality education, research, and consultancy by providing principled and highly skilled mathematics.

MASTER OF SCIENCE IN MATHEMATICS

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

1. Graduates will contribute rapidly growing multidisciplinary research that uses advanced computing capabilities to understand and solve complex problems.
2. Graduate of the programme will be capable of handling every problem existing around the world through mathematical structures.
3. Graduate of the programme will become competent users of mathematics and to provide mathematical solution to real life problems.
4. Graduates will continue lifelong learning and pursue higher studies in mathematical and statistical sciences

PROGRAM OUTCOME: After completing this programme graduate will be able to

a. Progress the critical analysis and problem – solving skills required for research and development organization and industry.

b. Communicate confidently and effectively with industry and society at large, regarding complex problem and solution of the problem, existing around.

c. Engage independent and lifelong learning with a high level of enthusiasm and commitment to improve knowledge and competence continuously.

d. Contribute significantly to academics through teaching and research.

e. Demonstrate knowledge and understanding of various structure of mathematics and apply the same to one's own work, as a member and leader in a team, manage projects efficiently after consideration of economic and financial factors.

f. Apply ethical principles and commit to professional ethics and responsibilities and norms of the professional practice.

BOARD OF STUDIES COMMITTEE

The board of studies for Master of Science in Mathematics of department of mathematics includes the following members:

Chairperson

- Dr. V. Lakshmana Gomathi Nayagam, Associate Professor and Head, Department of Mathematics

Coordinator

- Dr. Vamsinadh Thota, Assistant Professor, Department of Mathematics

External Experts

- Dr. Satyajit Roy, Professor, Department of Mathematics,
  Indian Institute of Technology Madras, Chennai, Tamil Nadu

- Dr. A. Raja, General Manager, AMAR Tech, Bangalore – 560004
  (Siemens Centre of Excellence, NIT Tiruchirappalli)

- Dr. A. Raghunathan, Additional General Manager, BHEL, Tiruchirappalli

Members

- Dr. R. Ponalagusamy, Professor (HAG)
- Dr. K. Murugesan, Professor (HAG)
- Dr. T. N. Janakiraman, Professor (HAG)
- Dr. V. Kumaran, Professor
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- Dr. P. Saikrishnan, Professor
- Dr. R. Tamil Selvi, Associate Professor
- Dr. V. Shanthi, Associate Professor
- Dr. I. Jeyaraman, Assistant Professor
- Dr. N. Prakash, Assistant Professor
- Dr. Jitraj Saha, Assistant Professor
- Dr. N. Shivaranjani, Assistant Professor
- Dr. M. Sivanesan, Assistant Professor
- Dr. Atul Kumar Verma, Assistant Professor
- Dr. Abhijit Das, Assistant Professor

COURSE CURRICULUM

SEMESTER – 1

S. No	Code	Course of study	L	T	P	Credit
1.	MA 701	**Real Analysis**	3	0	0	3
2.	MA 703	**Linear Algebra**	3	0	0	3
3.	MA 705	**Probability and Statistics**	3	0	0	3
4.	MA 707	**Ordinary Differential Equations**	3	0	0	3
5.	MA 709	**Programming language - C**	3	0	0	3
6.	MA 711	**Programming in C-lab**	0	0	4	2
					Total	**17**

SEMESTER – 2

S. No	Code	Course of study	L	T	P	Credit
1.	MA 702	**Algebra**	3	0	0	3
2.	MA 704	**Complex Analysis**	3	0	0	3
3.	MA 706	**Topology**	3	0	0	3
4.	MA 708	**Partial Differential Equations**	3	0	0	3
5.	MA 710	**Numerical Analysis**	3	0	0	3
6.	MA 712	**Statistical Computing-Lab**	0	0	4	2
					Total	**17**

SEMESTER – 3

S. No	Code	Course of study	L	T	P	Credit
1.	MA 713	**Functional Analysis**	3	0	0	3
2.	MA 715	**Transforms Techniques**	3	0	0	3
3.	MA 717	**Operations Research**	3	0	0	3
4.	** ** **	**Elective - 1**	3	0	0	3
5.	** ** **	**Elective – 2**	3	0	0	3
6.	MA 719	**Mathematical software Lab**	0	0	4	2
					Total	**17**

SEMESTER – 4

S. No	Code	Course of study	L	T	P	Credit
1.	** ** **	**Elective 3**	3	0	0	3
2.	** ** **	**Open Elective/Elective 4**	3	0	0	3
3.	MA 714	**Project**	0	0	0	8
					Total	**14**

TOTAL CREDITS: 65

LIST OF ELECTIVES

S. No	Code	Course of study	L	T	P	Credit
1.	MA 718	Operator Theory	3	0	0	3
2.	MA 720	Fuzzy Mathematics and its Applications	3	0	0	3
3.	MA 721	Graph Theory	3	0	0	3
4.	MA 723	Measure Theory	3	0	0	3
5.	MA 724	Integral Equations & Calculus of Variations	3	0	0	3
6.	MA 726	Non – Linear Programming	3	0	0	3
7.	MA 727	Numerical Solution of DE	3	0	0	3
8.	MA 729	Fluid Dynamics	3	0	0	3
9.	MA 802	Introduction to Fuzzy Mathematics and its Applications	3	0	0	3
10.	MA 803	Advanced Fuzzy Mathematics and its Applications	3	0	0	3
11.	MA 804	Advanced Numerical Computations	3	0	0	3
12.	MA 805	Advanced Numerical Analysis on Differential Equations	3	0	0	3
13.	MA 806	Matrix Theory and Stochastic Programming	3	0	0	3
14.	MA 850	Advanced Numerical Analysis for Singularly Perturbed Differential Equations	3	0	0	3
15.	MA 852	Biorheology	3	0	0	3
16.	MA 854	Introduction to Singularly Perturbed Equation and Differential Equations	3	0	0	3
17.	MA 855	Advanced Complex Analysis	3	0	0	3
18.	MA 856	Geometric Function Theory	3	0	0	3
19.	MA 857	Approximation Theory	3	0	0	3
20.	MA 858	Convex Analysis	3	0	0	3
21.	MA 859	Matrix Analysis	3	0	0	3
22.	MA 860	Optimization Techniques	3	0	0	3
23.	MA 861	Hardy – Hilbert Spaces	3	0	0	3
24.	MA 862	Asymptotic and Perturbation Methods	3	0	0	3
25.	MA 863	Sobolev Spaces and its Applications	3	0	0	3
26.	MA 864	Graphs and Matrices	3	0	0	3
27.	MA 865	Applied Functional Analysis	3	0	0	3
28.	MA 866	Particulate Processes: Theory and Modeling	3	0	0	3
29.	MA 867	Fixed Point Theory and its Applications	3	0	0	3
30.	MA 868	Advanced Functional Analysis	3	0	0	3
31.	MA 869	Theory and Geometry of Banach Spaces	3	0	0	3
32.	MA 870	Fitted Mesh and Fitted Operator Methods for Singular Perturbation Problems	3	0	0	3
33.	MA 871	Mathematical Theory of Water Waves	3	0	0	3
34.	MA 872	Introduction to Hydrodynamic Stability	3	0	0	3

Course Code	MA 701
Title of the Course	REAL ANALYSIS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: This course

1. introduces various concepts related to real numbers, differentiation, and integration.
2. also deals with mean value theorems and convergence of series of functions. In addition, an introduction to metric spaces is also given.

Course Content

Infimum, supremum, and limit point of a subset of real numbers. liminf, limsup and limit of a sequence of real numbers. Nature of series of real numbers. Limit, continuity, differentiation, and Riemann integration of real valued functions. Riemann – Stieltjes Integral, existence of the integral. Condition for integrability, properties, integral as a limit of a sum, first mean value theorem, Second mean value theorem. The Riesz representation theorem.

Sequences and series of real valued functions, pointwise convergence, uniform convergence, Cauchy's criterion, and test for uniform convergence of sequence of functions. Tests for uniform convergence of series of functions (Weierstrass's M – test, Abel's test, Dirichlet's test). Uniform convergence versus continuity (Dini's theorem), integration and differentiation. The Weierstrass approximation theorem.

Metric spaces, basic concepts, Cauchy's sequence, and convergence of a sequence in metric spaces. Complete metric spaces. Connectedness, intermediate value theorem. Separable metric spaces. Compactness, Heine – Borel theorem. Continuous and uniformly continuous functions from one metric space to other. The Banach Contraction Principle. Continuous functions on a metric space. Homeomorphisms, Equivalent metrics. Completion of a metric space. Equi – continuous family of functions. The Arzela – Ascoli theorem, The Baire Category Theorem.

Reference Books:

1. N. L. Carothers, Real Analysis, Cambridge University Press, 2000
2. H. L. Royden, P. M. Fitzpatrick, Real Analysis, 4th ed., Pearson education, 2011
3. W. Rudin, Principles of Mathematical Analysis, Mc-Graw Hill, 1976
4. G. F. Simmons, Introduction to and Modern Analysis, Kreiger Publishing Co., 1983

Course Learning Outcomes: Completion of the course, students will be to

1. find liminf, limsup and discuss continuity and differentiability of functions
2. define Riemann- Stieltjes integral, evaluate them for various functions and understand their properties.
3. test convergence of series of functions.
4. understand the various concepts of topology in the metric space setting.

Course Code	MA 703
Title of the Course	LINEAR ALGEBRA
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. discuss various decompositions of vector spaces and linear transformations on vector spaces.
2. study diagonalizable operator on a vector space and characterizations of it using the minimal and characteristic polynomials.
3. introduce different classes of linear operators on inner product spaces and to study their structures.
4. learn the concepts of bilinear and quadratic forms on vector spaces.

Course Content

Review of basic concepts: Vector spaces, Bases, Dimension, Linear Transformations - The matrix representation – Change of basis – Rank and Nullity – System of linear equations.

Characteristic values and characteristic vectors – Eigen spaces - Algebraic and Geometric Multiplicities – Diagonalization –– Minimal polynomial – Cayley-Hamilton Theorem.

Invariant subspaces – Direct-sum Decompositions – Invariant Direct sums – The Primary Decomposition Theorem – The Jordan Canonical form.

Basic review of Inner Product Spaces – Adjoint operators – Normal operators – Unitary Operators – Orthogonal projections – The spectral Theorem.

Bilinear forms – Matrix representation – Quadratic forms – Positive definite forms – Sylvester's law of inertia.

Reference Books:

1) Kenneth Hoffman and Ray Kunze, "*Linear Algebra*", PHI, 2010.
2) Stephen H. Friedberg, Arnold J. Insel and Lawrence E. Spence, "*Linear Algebra*", PHI, 2013.
3) Sheldon Axler, "*Linear Algebra Done Right",* Springer, 1997.
4) Steven Roman, "A*dvanced Linear Algebra",* Springer, 2008.

Course Learning Outcomes: On completion of the course, the students will be able to

1. find characteristic values, characteristic vectors, and the minimal polynomial of a linear transformation and to determine a linear transformation is diagonalizable.
2. decompose a vector space into a sum of invariant subspaces and a linear transformation into a direct sum of induced operators.
3. compute the cyclic subspace generated by a vector and to construct the rational and Jordan forms of linear transformations and matrices.
4. determine a linear operator is normal, unitary, and orthogonal projection and to construct the spectral decomposition of normal and self-adjoint operators.
5. construct the matrix of a bilinear form and to find index, rank, and signature of a bilinear form.

Course Code	MA 705
Title of the Course	PROBABILITY AND STATISTICS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

- understand and use the probability concepts in formulating and study real life situation.
- provide a solution for real life problems using the various elements probability, probability density function, moments, probability calculation etc.
- apply the concepts of estimation theory, random sampling, test statistical hypotheses in solving many real – life applications for further improvement and modification.

Course Content

Definitions of probability - Probability spaces- Random variables- Probability Mass and Density functions- Discrete and Continuous distributions - Standard and non-standard types.

Mathematical expectation- Generating functions- Probability and Moment generating functions- Characteristic function – Two variables - Joint distribution function -Conditional density and Expectations – Covariance - Coefficient of correlation - Multiple random variables and its moments.

Chebyshev's - Markov type inequalities- Convergences in probability– Law of large numbers- Central limit theorem- Applications.

Fundamental concepts in statistics- Measures of location and variability- Population, sample, parameters- Point and interval estimation- Method of moments, Maximum likelihood estimator, Properties of estimator, Unbiasedness, Consistency, Efficiency- Confidence intervals for mean, Difference of means, Proportions.

Testing of hypothesis - Null and alternate hypothesis -Neyman Pearson fundamental lemma - Tests for one sample and two sample problems for normal populations - Tests for proportions - Test for small samples – t - test, chi - square test and F-test.

Reference Books:

1. William Feller: *An Introduction to Probability Theory and its Applications,* 3rd edition, Vol. I and Vol II, New York, Wiley India, 2008.
2. Kai Li Chung: *A course in Probability Theory,* 3rd edition, Academic Pres, 2001.
3. S. M. Ross: *Introduction to Probability Models*, 11th edition, Academic Press, 2014.
4. Robert V. Hogg, J.W. McKean, and Allen T. Craig: *Introduction to Mathematical Statistics*, 7th Edition, Pearson Education, Asia, 2014.
5. Edward J Dudewicz and Satya N. Mishra: *Modern Mathematical Statistics*, International Edition, Wiley. 1988.

Course Learning Outcomes: Completion of the course, student will be to

1. understand the axiomatic rudiments of modern Probability theory and use of random variables as an intrinsic tool for the analysis of random phenomena.
2. characterize multiple input and output system probability models and function of random variables based on single & multiple random variables.
3. evaluate and apply moments & characteristic functions and understand the concept of inequalities and probabilistic limits.
4. able to use basic statistical knowledge in testing hypotheses on large and small samples and estimations.

Course Code	MA 707
Title of the Course	ORDINARY DIFFERENTIAL EQUATIONS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: This course introduces

1. various approach to find general solution of the ordinary differential equations
2. theorems to discuss the existence and uniqueness of solution of IVP for ODE
3. special functions and its properties

Course Content

The general solution of the homogeneous equation –The method of variation of parameters – Power Series solutions- Higher order linear equation-operator methods for finding particular solutions.

Series solutions of first order equations – Second order linear equations; Ordinary points. Regular Singular Points – Gauss's hypergeometric equation – The Point at infinity - Legendre Polynomials – Bessel functions – Properties of Legendre Polynomials and Bessel functions.

Linear Systems of First Order Equations – Homogeneous Equations with Constant Coefficients – The Existence and Uniqueness of Solutions of Initial Value Problem for First Order Ordinary Differential Equations – The Method of Solutions of Successive Approximations and Picard's Theorem.

Oscillation Theory and Boundary value problems – Qualitative Properties of Solutions – Sturm Comparison Theorems – Eigenvalues, Eigen functions and the Vibrating String.

Nonlinear equations: Autonomous Systems; the phase plane and its phenomena – Types of critical points; Stability – critical points and stability for linear systems – Stability by Liapunov's direct method – Simple critical points of nonlinear systems.

Reference Books:

1. E.A. Coddington, An Introduction to Ordinary Differential Equations, Courier Corporation, 2012
2. G.F. Simmons, Differential Equations with Applications and Historical Notes, CRC Press, 2016
3. M.E. Taylor, Introduction to Differential Equations, AMS Indian Edition, 2011.
4. William E. Boyce, Richard C. Di Prima, Douglas B. Meade, Elementary Differential Equations and Boundary Value Problems, Wiley, 2017.
5. Lawrence Perko, Differential Equations and Dynamical Systems, Springer Science & Business Media, 2013

Course Learning Outcomes: Completion of the course, student will be able to

1. find the solutions of first and some higher order ordinary differential equations
2. discuss the existence and uniqueness of solutions of first and second order ODE
3. apply properties of special functions in discussion the solution of ODE.
4. model some physical problem and give physical interpretation of the solution.

Course Code	MA 709
Title of the Course	PROGRAMMING IN C
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: This course makes the student to

1. understand and write programs in language of C for a given problems.
2. analyze the concepts of arrays and tables for storage
3. involve in creating files for the problems.
4. interpret the programs through pointers.

Course Content

Introduction to C: The C character set - Identifiers, Constants, and keywords -Primitive datatypes - Operators and Expressions-Library functions- Data Input and Output.

Control Statements: Nested control structures - Functions-Function prototypes –Passing arguments to a function.

Program Structure: Storage classes –Arrays-Declaration, initialization, and accessing array elements-Arrays and strings.

Pointers: Pointer declarations -Passing pointers to a function -Pointers and one-dimensional arrays - Dynamic memory allocation -Operations on pointers -Pointers and multidimensional arrays -Arrays of pointers -Passing functions to other functions.

Structures and Unions: Defining a structure -Processing a structure -User-defined datatypes(typedef) -Structures and pointers -Passing structures to functions -Self-referential structures. Data Files-Operations-Formatted input and output- Character input and output.

Reference Books:

1. B. S. Gottfried & J. K. Chhabra, Programming with C, Second Edition, Tata McGraw- Hill, New Delhi, 2006.
2. B.W. Kernighan & D. M. Ritchie, The C Programming Language, Second Edition, Prentice Hall of India Pvt. Limited, New Delhi, 2006.
3. V. Rajaraman, Computer Programming in C, Prentice Hall of India Pvt. Ltd. New Delhi, 2004.
4. E. Balagurusamy, Programming in ANSI C by, 7^{th} Edition, Tata McGraw-Hill Publishing Co. Ltd., New Delhi, 2017.

Course Learning Outcomes: Completion of the course, student will be able to

1. write structured pseudo codes for a given problem.
2. design programs in C for any given problem.
3. manage writing programs for complex problems.
4. attain the capability of developing files through pointers.

Course Code	MA 711
Title of the Course	PROGRAMMING IN C – LABORATORY
Prerequisite	Nil
Credits (L-T-P)	2 (0 – 0 – 2)

Course Learning Objectives: This course to

1. make the student learn a programming language.
2. learn problem solving techniques.
3. teach the student to write programs in C and to solve the problems.

Course Content

Implementing the concepts of programming language C: Basics, operators, Loop operations, Arrays, Math Functions and I/O Functions, Functions, Functions and Recursion, Structures, File operations using command line arguments.

Reference Books:

1. B. S. Gottfried & J. K. Chhabra, Programming with C, Second Edition, Tata McGraw- Hill, New Delhi, 2006.
2. B.W. Kernighan & D. M. Ritchie, The C Programming Language, Second Edition, Prentice Hall of India Pvt. Limited, New Delhi, 2006.
3. V. Rajaraman, Computer Programming in C, Prentice Hall of India Pvt. Ltd. New Delhi, 2004.
4. E. Balagurusamy, Programming in ANSI C by, 7^{th} Edition, Tata McGraw-Hill Publishing Co. Ltd., New Delhi, 2017.

Course Learning Outcomes: Completion of the course, student will be able to

1. read, understand, and trace the execution of programs written in C language.
2. write the C code for a given algorithm.
3. develop programs for complex problems applying the concepts of Arrays and pointers.

Course Code	MA 702
Title of the Course	ALGEBRA
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. introduce the concepts of conjugacy classes and Sylow's theorems
2. explain the Fundamental Theorem of Finite Abelian Groups
3. learn the various types of integral domains
4. expose the students to extensions field and its properties
5. learn the Galois Theory and solvability.

Course Content

Review of basic Group Theory – Group actions – Conjugacy classes – The class equation – Sylow's Theorem - Direct Product –Fundamental Theorem of Finite Abelian Groups.

Review of basic Ring Theory – Ideals and Factor rings – Prime and Maximal ideals– Euclidean domains– principal ideal domains and unique factorization domains–Polynomial rings – Factorization of Polynomials.

Extension fields – Splitting fields – Algebraic and Transcendental extensions – Simple extensions – Separable extensions - Finite fields.

Galois Theory – Fundamental Theorem of Galois Theory – Solvability of Polynomials by Radicals – Solvable groups – Insolvability of a quantic.

Reference Books:

1. D. S. Dummit and R. M. Foote: *Abstract Algebra*, 3rdEdition, John-Wiley, 2011.
2. M. Artin: *Algebra*, 2nd edition, Pearson, 2011.
3. I.N. Herstein: *Topics in Algebra*, 2nd edition, John-Wiley, 2008.
4. J.A. Gallian: *Contemporary Abstract Algebra*, 4thedition, Narosa, 1999.
5. N. Jacobson: Basic Algebra I and II, 2nd Edition, Dover Publication Inc., 2009.

Course Learning Outcomes: Completion of the course, student will be able to

1. analyze the concepts of conjugacy classes and Sylow's theorem
2. understand the properties of various type of integral domains
3. gain the knowledge on the extension fields
4. understand the concepts of Galois Theory and solvability

Course Code	MA 704
Title of the Course	COMPLEX ANALYSIS
Prerequisite	MA 701
Credits (L-T-P)	3(3 – 0 – 0)

Course Learning Objectives:

The course presents an introduction to analytic functions, conformal mappings, Mobius transformations and power series. Various Cauchy' theorems are discussed and used in evaluation of integral. It deals with locations of zeros of analytic functions and maximum principles.

Course Content

Lines and planes in complex plane, extended complex plane, spherical representation, power series, analytic functions as mappings, branch of logarithm, conformal mappings, Mobius transformations.

Power series representation of analytic functions, zeros of analytic functions, index of a closed curve, Cauchy's theorem and integral formula on open subsets of C.

Homotopy, homotopic version of Cauchy's theorem, simple connectedness, counting of zeros, open mapping theorem, Goursat's theorem, Classification of singularities, Laurent series.

Residue, Contour integration, argument principle, Rouche's theorem, Maximum principle, Schwarz' lemma.

Reference Books:

1. Conway John. Functions of One Complex Variables. 2nd ed, Narosa, New Delhi. 2002.
2. Ahlfors Lars. Complex Analysis. McGraw Hill Co., New York. 1988.
3. Hahn Liang-Shin and Epstein Bernard. Classical Complex Analysis. Jones andBartlett India, New Delhi. 2011.
4. Rudin Walter. Real and Complex Analysis. McGraw-Hill. 1987.
5. Ullrich David. Complex Made Simple. American Math. Soc., Washington DC. 2008.

Course Learning Outcomes: Completion of the course, student will be able to

1. understand analytic functions as mappings and discuss properties of conformal mappings, and Mobius transformations
2. obtain series representation of analytic functions
3. evaluate various integrals by using Cauchy's residue theorem
4. classify singularities and derive Laurent series expansion

Course Code	MA 706
Title of the Course	TOPOLOGY
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: This course

1. introduces the notion of open and closed sets and deals with continuity, connectedness, compactness and various countability and separation axioms.

Course Content

Finite, countable, uncountable sets. Functions and relations. Partially ordered sets, well ordered sets. Axiom of choice, Well-ordering theorem, The maximum principle, Zorn's lemma.

Topological spaces, open sets, closed sets. closure and interior of a set. Limit points of a set. Basis for a topology, sub basis. subspace topology, order topology, product topology $X \times Y$. T1, T2 and Hausdorff spaces, Metric topology.

Continuous functions, Homeomorphisms, constructing continuous functions, The pasting lemma. Product topology, box topology, quotient topology.

Connected spaces, components and locally connectedness, path connectedness. Compact spaces, limit point compactness, sequentially compactness, local compactness, finite intersection property. Compactifications.

Countability axioms, separation axioms, regular and normal spaces. The Urysohn's lemma, The Urysohn's metrization theorem, The Tietze extension theorem, The Tychonoff theorem.

Reference Books:

1. J. R. Munkres, Topology, 2nd ed., Pearson Education India, 2001.
2. G. F. Simmons, Introduction to Topology and Modern Analysis, Kreiger Publishing Co., 1983
3. C. Wayne Patty, Foundations of Topology, Jones and Batlett, Delhi 2010.

Course Learning Outcomes: Completion of the course, student will be able to

1. understand various notions of topological spaces and derived concepts.
2. prove results about homeomorphism, product topology, connectedness, and compactness.
3. prove theorems about Hausdorff spaces, Regular and Normal spaces.
4. construct new continuous functions and prove compactness in arbitrary product spaces.

Course Code	MA 708
Title of the Course	PARTIAL DIFFERENTIAL EQUATIONS
Prerequisite	MA 707
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: This course

1. discuss various approach to find the solution of partial differential equations.
2. construct mathematical model and solution of some physical problem

Course Content

FIRST ORDER EQUATIONS Integral surfaces passing through a given curve - Surfaces orthogonal to a given system of surfaces - Compatible system of equations - Charpit's method.

SECOND ORDER EQUATIONS Classification of second order Partial Differential Equations - Reduction to canonical form - Adjoint operators.

HYPERBOLIC EQUATIONS One-dimensional wave equation - Initial value problem - D' Alembert's solution - Riemann - Volterra solution - Vibrating string - Variables Separable solution - Forced vibrations - Solutions of Non-homogeneous equation - Vibration of a circular membrane.

PARABOLIC EQUATIONS Diffusion equation - Method of Separation of variables: Solution of one- and two-dimensional Diffusion equations in Cartesian coordinates and Solution of Diffusion equation in cylindrical and spherical polar coordinates.

ELLIPTIC EQUATIONS Boundary value problems - Properties of harmonic functions - Green's Function for Laplace Equation - The Methods of Images - The Eigen function of Method.

Reference Books:

1. I. N. Snedden, Elements of Partial Differential Equations, McGraw Hill, 1985.
2. T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Publishing Company, 1997.
3. Phoolan Prasad and Renuka Ravindran, Partial Differential Equations, Wiley-Eastern Ltd, 1985.
4. Lokenath Debnath and Dambaru Bhatta , Integral Transforms and Their Applications, Chapman & Hall/CRC; 2 edition, 2006.
5. Tyn Myint-U: Partial differential equations for scientists and engineers, 3rd ed. North Holland, 1989.
6. I.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19 AMS, 1998.

Course Learning Outcomes: Completion of the course, student will be able to

1. form the partial differential equation for family of surfaces.
2. find solution of Laplace equation for various boundary conditions.
3. model vibration of an elastic string/membrane and find discuss solution of it.
4. model one dimensional heat equation and find analytic solution for some boundary condition.

Course Code	MA 710
Title of the Course	NUMERICAL ANALYSIS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. introduce various numerical algorithm to find numerical solution of mathematical equation.
2. validate numerical solution through mathematical analysis.

Course Content

Error Analysis: Exact and approximate numbers, rounding of numbers, Significant digits, Correct digits, various types of errors encountered in computations, Propagation of errors.

Solution of system of linear equations: (i) Direct methods: Gauss elimination method, Gauss Jordon method, LU- decomposition method. (ii) Iterative methods: Jacobi and Gauss-Seidel methods.

Eigen values and Eigen vectors: Dominant and smallest Eigen values/Eigen vectors by power method.

Roots of non-linear equations: Bisection method, Regula-Falsi method, Newton-Raphson method, direct iterative method with convergence criteria, Graeffe's root squaring method, Newton-Raphson method for solution of a pair of non-linear equations.

Interpolation: Finite difference operator and their relationships, difference tables, Newton, Bessel and Stirling's interpolation formulae, Divided differences, Lagrange interpolation and Newton's divided difference interpolation.

Numerical differentiation: First and second order derivatives by various interpolation formulae.

Numerical integration: Trapezoidal, Simpsons 1/3 and 3/8 rules with errors and their combinations, Gauss Legendre 2-points and 3-points formulae

Solution of first and second order ordinary differential equations: Taylor's series method, Euler, Modified Euler, Picard's method, Runge – Kutta methods and Milne's and Adam's method. Stability Analysis for single-step and multi-step method

Reference Books:

1) David Kinciad & Ward Cheney, Numerical Analysis and mathematics of scientific computing, Brooks/Cole, 1999
2) K. Atkinson, Elementary Numerical Analysis, Jhon Wiley & Sons, 2004
3) Curtis E Gerald & Partrock O Whealtley, Applied Numerical Analysis, Pearson edu. 2004
4) M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Method: For Scientific and Engineering Computation, New Age International, 7thedn, 2019
5) John H. Mathews, Kurtis K. Fink, Numerical Methods Using MATLAB, 4thEdn, Pearson, 2004

Course Learning Outcomes: Completion of the course, student will be able to

1. find the numerical solution of linear system of equations AX=b
2. find the roots of transcendental and polynomial equations
3. approximate the function and interpolate function and its derivatives
4. find numerical differentiation of the function and numerical solution of ODEq.
5. find single and double integral numerically.

Course Code	MA 712
Title of the Course	STATISTICAL COMPUTING – LABORATORY
Prerequisite	Nil
Credits (L-T-P)	2 (0 – 0 – 2)

Course Learning Objectives: Objective of the course is to

1. make the student learn statistical software and perform several statistical tests using R.
2. use built-in function to construct correlation and regression of given data.
3. perform statistical analysis over the large data.

Course Content

Overview of R, R data types and objects, reading and writing data, Control structures, functions, scoping rules, dates and times, Loop functions, debugging tools, Simulation, code profiling.

Reference Books:

1. Kun Ren, Learning R Programming, Packt Publishing Ltd, 2016.
2. Colin Gillespie and Robin Lovelace, Efficient R Programming: A Practical Guide to Smarter Programming, "O'Reilly Media, Inc.", 2017

Course Learning Outcomes: Completion of the course, student will be able to

1. find statistical parameters (mean, median, etc.) for given lager data
2. find correlation coefficient between two variables,
3. find regression line & regression curve for large data
4. present graphical representation and analyze the data

Course Code	MA 713
Title of the Course	FUNCTIONAL ANALYSIS
Prerequisite	MA 701, MA 706
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course, is to

1. exposes the students to normed spaces, Banach spaces and Hilbert space by discussing the concepts of compactness, bounded linear operators.
2. introduces the various notions used in inner product space.
3. Introduces Hahn-Banach theorems, open mapping theorem, closed graph theorem, Riesz representation theorem are proved.

Course Content

Normed linear spaces. Metric induced by the norm. Cauchy's sequence, convergence of a sequence. Continuity of vector space operations and norm. Types of convergence of a series. Banach spaces. Finite dimensional normed linear spaces and subspaces. Equivalent norms and their properties. Compactness and finite dimension. Quotient space and products of normed linear spaces.

Bounded linear transformations. Space of Bounded linear transformations B[X, Y] and its properties. Continuous linear functionals and Duals of classical spaces. The Hahn-Banach extension and separation theorems. The open mapping theorem and its applications (The inverse mapping and closed graph theorems). The principle of uniform boundedness (Banach-Steinhaus theorem).

Inner product spaces. Norm induced by the inner product. Schwartz inequality, Parallelogram identity. Hilbert spaces. Relation between Banach and Hilbert spaces. Closest point in a closed convex subset. Projection theorem. Orthogonal complement of a subspace, Orthogonal decomposition. Orthogonal and orthonormal systems. Bessel's inequality, Parseval's identity. Characterizations of complete orthonormal systems. The Riesz representation theorem.

Reference Books:

1. B. Bollabas, Linear Analysis, Cambridge University Press (Indian edition),1999
2. J. B. Conway, A Course in functional Analysis, 2nd ed., Springer, 1985
3. E. Kreyszig, Introduction to Functional Analysis with Applications, Wiley, 1989
4. G. F. Simmons, Introduction to Topology and Modern Analysis, Krieger Publishing Co., 1983
5. A. E. Taylor and D. C. Lay, Introduction to Functional Analysis, 2nd ed., Wiley, New York, 1980.

Course Learning Outcomes: Completion the course, student will be able to

1. understand various concepts of normed spaces and construct new spaces from old ones.
2. find duals of standard spaces and also prove fundamental results about bounded linear functional.
3. find orthogonal decomposition of Hilbert spaces and prove several other results.
4. understand basics of various operators and their properties.

Course Code	MA 715
Title of the Course	TRANSFORMS TECHNIQUES
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to
1. introduce various transform technique to solve mathematical equation representing engineering problem.
2. discuss the properties of various integral transforms.
3. express periodic and non-periodic function in terms of sinusoidal functions.

Course Content

LAPLACE TRANSFORMS-Transforms of elementary functions - Properties - Differentiation and integration of transforms - Periodic functions - Initial & final value theorems - Inverse Laplace transforms - Convolution theorem - Error function - Transforms involving Bessel functions.

FOURIER SERIES: Dirichlet's Conditions – General Fourier Series – Half Range Sine Series – Half Range Cosine Series – Complex Form of Fourier Series – Parseval's Identity – Harmonic Analysis.

FOURIER TRANSFORMS: Fourier integral representation - Fourier transform pairs - Properties - Fourier sine and cosine transforms - Transforms and inverse transforms of elementary functions - Convolution theorem - Transforms of derivatives.

APPLICATIONS OF TRANSFORMS: Application of Laplace Transforms - Evaluation of integrals - Solution of Linear ODE - Applications of Fourier Transforms - Heat equation on infinite and semi-infinite line - Potential problems in half-plane.

Z- TRANSFORMS AND DIFFERENCE EQUATIONS: Z-Transforms – Elementary Properties – Inverse Z – Transform (Using Partial Fraction and Residues) – Convolution Theorem – Formation of Difference Equations – Solution of Difference Equations Using Z – Transform.

Reference Books:

1. Andrews, L.C. and Shivamoggi, B.K., "Integral Transforms for Engineers", SPIE press, 1999.
2. Sneddon, I.N., Fourier Transforms, Courier Corporation, 1995.
3. Andrews, L.C. and Shivamoggi, B.K., "Integral Transforms for Engineers and Applied Mathematicians", Mac Millan Publishing Co., 1988.
4. LokenathDebnath, Dambaru Bhatta, Integral Transforms and Their Applications, 3^{rd}edn, CRC Press, 2016
5. Anthony C. Grove, An Introduction to the Laplace transform and the Z-transform, Prentice Hall, 1991.

Course Learning Outcomes: Completion of course, student will be able to
1. find Laplace transform of functions and discuss various properties of Laplace transforms
2. express periodic functions in terms of sinusoidal functions
3. find Fourier transform of functions and discuss various properties of Fourier transforms
4. find Z-Transform transform of discrete functions and discuss various properties of Z-transforms
5. Apply transform to solve differential and difference equations.

Course Code	MA 717
Title of the Course	OPERATIONS RESEARCH
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course, is to

1. formulate various real-life problems as Operations Research models and to study methodologies to solve the problems.
2. Introduce Linear Programming, Transportation and Assignment problems and to discuss methods to find optimum solutions.
3. study the theory of duality and sensitivity analysis in linear programming.
4. learn network flow problems and their solution techniques.
5. explore dynamic programming problem and its applications.

Course Content

Introduction – Models in Operations Research – Linear Programming Problems – Simplex Method – Big-M Method – Two-Phase Method – Special Cases – Degeneracy and Cycling – Unbounded Solutions – Alternative Optima.

Dual Linear Programs – Duality Theorems – Dual Simplex Method - Transportation Problems – Finding an Initial Basic Feasible Solution - Optimality Condition – MODI method – Degeneracy – Assignment Problems – Hungarian Method.

Revised Simplex Method – Sensitivity Analysis – Parametric Programming.

Network Analysis – Shortest Route Problems – Maximal Flow Problems – Critical Path Method (CPM) – Program Evaluation and Review Techniques (PERT).

Dynamic Programming – Introduction – Principle of Optimality – Forward and Backward recursions – Discrete Dynamic Programming – Continuous Dynamic Programming – Applications.

Reference Books:

1. A. Ravindran, Don T. Phillips and James J. Solberg, *Operations Research- Principles and Practice*, John Wiley, 2014.
2. Hamdy A. Taha, *Operations Research-An Introduction*, Prentice Hall of India, 2000.
3. Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill, 2010.
4. KantiSwarup, P.K. Gupta and Man Mohan, *Operations Research*, Sultan Chand, 2014.

Course Learning Outcomes: On completion of the course, the students will be able to

1. solve Linear Programming Problem (LPP) using Simplex, Big-M and Two-phase methods.
2. find an optimum solution for transportation and assignment problems and to analyze LPP using duality results.
3. solve LPP using Revised Simplex method and to apply duality methods in the study of sensitivity analysis in LPP and parametric programming.
4. determine the shortest path, critical path, and maximal flow in a network.

Course Code	MA 719
Title of the Course	MATHEMATICAL SOFTWARE – LABORATORY
Prerequisite	Nil
Credits (L-T-P)	2 (0 – 0 – 2)

Course Learning Objectives: Objective of the course, is to

1. introduce software available to solve mathematical problem.
2. understand and practice basic operators, functions available in SCILAB,
3. Perform symbolic calculations using SCILAB.
4. get hands-on-training in LATEX and learn drawing using LATEX.

Course Content

Introduction to SCILAB, Scalars & Vectors, Matrix operations, polynomials, plotting, Functions, and loops. String Handling Functions. Basic programming in Scilab

Elements of LaTeX; Hands-on-training of LaTex; graphics in LaTeX; PSTricks; Beamer presentation;

Reference Books:

1. L. Lamport. LATEX: A Document Preparation System, User's Guide and ReferenceManual. Addison-Wesley, New York, second edition, 1994.
2. David F. Griffiths, Desmond J. Higham, Learning LaTeX: Second Edition, SIAM, 2016.
3. Sandeep Nagar, Introduction to Scilab: For Engineers and Scientist, Apress, 2017
4. Tejas Sheth, Scilab: A Practical Introduction to Programming and Problem Solving, CreateSpace Independent Publishing Platform, 2016.

Course Learning Outcomes: On completion of the course, the students will be able to

1. perform basic operations and symbolic calculations in SCILAB.
2. Draw two- and three-dimensional graphs using SCILAB.
3. use in-built functions to solve problem in ODE, Linear algebra, trigonometry.
4. make documents, presentation and draw geometries using LATEX.

MA 714 PROJECT WORK

Project work: Project work has 8 credits. The duration of the project work is one semester. Candidates can do the project work on recent research problem or latest research articles published in reputed international journals. The work can be done in the department itself or candidates can go to reputed department/ institution which has MOU with NITT.

Course Code	MA 718
Title of the Course	OPERATOR THEORY
Prerequisite	MA 701, MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: This course

1. introduces the dual representation of classical Banach spaces and necessary notions of convergence in Banach spaces.
2. also gives a detailed exposure of various operators on Banach spaces and Hilbert spaces.

Course Content

Dual space considerations: Representation of duals of the spaces c_00 with p-norms, c_0 and c with supremum-norm, l_p, C[a,b] and L_p. Reflexivity, weak and weak* convergences.

Adjoint, self-adjoint, normal and unitary operators on Hilbert spaces and their properties. Projection operators on Banach spaces and Hilbert spaces. Eigenvalues, Eigenvectors and Eigen spaces, Invariant spaces, Spectral theorem on finite dimensional Hilbert spaces.

Operators on Banach and Hilbert spaces: Compact operators and its properties; Integral operators as compact operators; Adjoint of operators between Hilbert spaces; Self-adjoint, normal and unitary operators; Numerical range and numerical radius; Hilbert-Schmidt operators.

Spectral results for Banach and Hilbert space operators: Eigen spectrum, Approximate eigen spectrum, Spectrum and resolvent; Spectral radius formula; Spectral mapping theorem; Riesz-Schauder theory, Spectral results for normal, Self-adjoint and unitary operators; Functions of self-adjoint operators.

Reference Books:

1. J. B. Conway, A Course in Functional Analysis, 2nd ed., Springer, 1985
2. J. B. Conway, A Course in Operator Theory, Graduate studies in Mathematics, vol. 21, AMS, 2000
3. R. G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics, vol.179, Springer-Verlag, 1998.
4. E. Kreyszig, Introduction to Functional Analysis with Applications, Wiley, 1989.

Course Learning Outcomes: Completion of the course, student will be able to

- analyze various convergence notions in Banach spaces.
- find the eigen spectrum, resolvent and spectral radius of operators.
- prove the spectral results of specific kind of operators.
- find representation of compact self-adjoint operators.

Course Code	MA 720
Title of the Course	FUZZY MATHEMATICS AND ITS APPLICATIONS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:
To enable the students to understand the concept of fuzzy logic, fuzzy sets, properties of α-cuts, extension principles, fuzzy complements, fuzzy intersection, fuzzy union, fuzzy numbers, fuzzy relations, fuzzy equivalence relations, classifications by fuzzy equivalence relations, c-means clustering, fuzzy c-mean clustering, fuzzy decision-making methods, and their applications.

Course Content

Fuzzy sets – introduction, Basic types and Basic concepts, Additional properties of □-cuts, Representation of fuzzy sets, Extension principles.

Type of operators on fuzzy sets and fuzzy complements, Fuzzy intersection and fuzzy unions, Combination of operations.

Fuzzy numbers and arithmetic operations on intervals, Arithmetic operations on fuzzy numbers, Fuzzy equations and fuzzy relations, Binary fuzzy relations and binary relations on a single set, Fuzzy equivalence relations.

Classification by equivalence relations-Crisp relations, Fuzzy relations, Cluster Analysis, Cluster Validity, c-means Clustering- Hard c-means (HCM), Fuzzy c-Means (FCM).

Fuzzy Decision making – introduction, Conversion of linguistic variables to fuzzy numbers, Individual Decision Making, Multi person decision Making, Multi criteria decision Making, Fuzzy ranking methods.

Reference Books:

1. George J. Klir, Bo Yuan, Fuzzy Sets and Fuzzy logic – Theory and Applications, Prentice Hall India, New Delhi, 1997.
2. H. J. Zimmermann, Fuzzy sets, Decision making and expert systems, Kluwer, Bosten, 1987.
3. S. J. Chen and C. L. Hwang, Fuzzy Multiple Attributes Decision Making, Springer verlag, Berlin Heidelberg, 1992.

Course Learning Outcomes: Upon successful completion of this course, students will be able to
1. analyze the types of fuzzy sets, α-cuts and its properties and extension of functions.
2. apply the operations (fuzzy complements, fuzzy intersections, and fuzzy unions) on fuzzy sets.
3. create the fuzzy relations and identify the different types of fuzzy relations and their applications.
4. apply the concepts of clustering of data (information) in engineering problems.
5. apply the concepts of fuzzy decision-making methods in engineering and management problems.

Course Code	MA 721
Title of the Course	GRAPH THEORY
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:

- To have general awareness of some applications-oriented concepts in Graph Theory and apply them as a tool in the various problems in different networks models.

Course Content

Introduction various types of graphs and applications- Definitions, examples and some results related to degree of vertex, walk, trail, path, tour, cycle traversals in a graph, complement of a graph, self-complementary graph, connectedness and connectivity graph, distance, shortest path in a graph, radius and diameter of a graph and various special graphs such as complete, cycles, bipartite, k-partite (with respect to some property). Some basic eccentric properties of graphs, tree, spanning tree, coding of spanning tree. Number of spanning trees in a complete graph. Recursive procedure to find number of spanning trees. Construction of spanning trees.

Directed graphs: Definitions and examples of strongly, weakly, unilaterally connected digraphs, strong components, and its applications. Matrix representation of graph and digraphs and some properties (proof not expected). Properties of Eulerian graphs and its applications, Hamiltonian graph-standard theorems, and applications (Dirac theorem, Chavtal theorem, closure of graph). Non-Hamiltonian graph with maximum number of edges. Self-centered graphs, dominating edge in a graph and some results related to eccentricity properties of complement of a graph and its applications.

Chromatic number; vertex chromatic number of a graph, edge chromatic number of a graph (only properties and examples)-applications to coloring. Planar graphs, Euler's formula, maximum number of edges in a planar graph, dual of planar embedding, some problems related to planarity and non-planarity, five color theorem(proof). Four color problem and its related results, Vertex Covering, Edge Covering, Vertex independence number, Edge independence number, relation between them and number of vertices of a graph.

Matching theory, maximal matching, algorithms for maximal matching and some results related to matching in bi-partite graphs and its applications. Perfect matching (only properties in general graph and applications to regular graphs). Some properties of Tournaments, some results on strongly connected tournaments. Notion of domination in graphs and various domination parameters in graph and their applications real life problems.

BFS and DFS algorithms and their applications, shortest path algorithm, Min-spanning tree and Max-spanning tree algorithms and their applications, Planarity algorithm. Flows in graphs; Maxflow-Mincut theorem, algorithm for maxflow. CPM in directed networks. Time complexity of all above polynomial time algorithms and other type problems; P-NP-NPC-NP hard problems and examples.

Reference Books:

1. J. A. Bondy and U. S. R. Murthy, Graph Theory with Applications, Macmillan, London (1976) EBook, Freely Downloadable.
2. Cormen, Leiserson, Rivest, and Stein, Introduction to Algorithms (Second edition), McGraw-Hill (2001).
3. M. Gondran and M. Minoux: Graphs and Algorithms, John Wiley, 1984.
4. Fred Buckley and Frank Harary, Distance in Graphs, Addison Wesley Publishing Company, Newyork, 1989.
5. T. W. Heynes, S. T. Hedetniemi, P.J.Slater, Fundementals of domination in graphs, Marcel Dekker, Inc. Newyork, 1998a.
6. T. W. Heynes, S. T. Hedetniemi, P. J. Slater, Domination in graphs, Advanced topics of domination in graphs, Marcel Dekker, Inc. Newyork, 1998b.

Course Learning Outcomes:

1. understand the various types of graphs, graph properties and give examples for the given property.
2. model the given problem from their field to underlying graph model.
3. proceed to solve the problem either through approximation algorithm or exact algorithm depending on the problem nature.
4. appreciate the applications of digraphs and graphs in various communication networks
5. appreciate the applications of graphs and digraphs in various other fields.

Course Code	MA 723
Title of the Course	MEASURE THEORY
Prerequisite	MA701
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:

This course exposes the students to Lebesgue measure and Lebesgue integral. It also introduces the convergence theorems involving integration, absolute continuity, and functions of bounded variation.

Course Content

Ring and algebra of sets, sigma-algebras. Borel algebra and Borel sets. Lebesgue outer measure on R, countable sub-additivity, measurable set, sigma-algebra structure of measurable sets-countable additivity of Lebesgue measure on R, Cantor set. Construction of a non-measurable subset of [0, 1], measurable functions, Approximation of measurable functions, Egorov's theorem, Lusin's theorem.

Lebesgue integral of non-negative measurable functions, Integrable functions and Lebesgue integral of integrable functions, linearity, Monotone convergence theorem, Fatou's lemma, Dominated convergence theorem. Comparison of Riemann and Lebesgue integration, Lebesgue integrability of Riemann integrable functions, characterization of Riemann integrable functions, improper Riemann integrals and their Lebesgue integrals.

Functions of bounded variation, Indefinite integrals of Lebesgue integrable functions on [a, b], statement of Vitali's lemma and almost everywhere differentiability of monotone increasing functions. Absolutely continuous functions, properties, absolute continuity of indefinite integral of Lebesgue integrable functions, Differentiation of indefinite integrals, characterization of absolutely continuous functions as indefinite integrals.

Completeness of L_p (μ) spaces. Signed measures, Hahn and Jordan decompositions, Radan-Nikodym theorem (without proof).

Reference Books:

1. G. de Barra, Measure and Integration, Wiley Eastern, 1981.
2. H. L. Royden, P. M. Fitzpatrick, Real Analysis, 4th ed., Pearson education, 2011
3. W. Rudin, Real and Complex Analysis, 3rd ed., McGraw-Hill International Editions, 1987.

Course Learning Outcomes: Completion of course, student will be able to

1. understand the concept of Lebesgue measure, measurable sets and approximation of a measurable function by simple measurable function.
2. prove various convergence theorems and know applications of them.
3. understand the absolutely continuous functions and functions of bounded variation.
4. do decomposition of measures.

Course Code	MA 724
Title of the Course	INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to introduce

1. different types of Integral equations arising in several engineering sectors, and methods/tools for solving different types of integral equations
2. the concept of resolvent kernels, Eigenvalues and eigenvectors of Fredholm integral equations, Green's function for a boundary value problem
3. necessity of calculus of variations in physical aspects (different problems and their method of solutions).
4. Galerkin and Collocation methods to obtain approximate solutions.

Course Content

Integral Equations: Basic concepts, Volterra integral equations, Relationship between linear differential equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equation of the first kind. Abel's integral equation. Fredholm integral equations, Fredholm equations of the second kind, The method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Eigenvalues and Eigen functions of a Fredholm alternative, Construction of Green's function for BVP, Singular integral equations.

Calculus of Variations: Euler – Lagrange equations, Degenerate Euler equations, Natural boundary conditions, Transversality conditions, Simple applications of variational principle, Sufficient conditions for extremum. Variational formulation of BVP, Minimum of quadratic functional. Approximate methods – Galerkin's method, Weighted-residual methods, Collocation methods. Variational methods for time dependent problems.

Reference Books:

1. David Porter, and David S.G. Stirling, *Integral Equations: A Practical Treatment, from Spectral Theory to Applications*, Cambridge texts in Applied Mathematics, 1990.
2. Ram P. Kanwal, *Linear Integral Equations: Theory and Technique*, Springer.
3. Abdul–Majid Wazwaz, *Linear and Non-linear Integral Equations: Methods and Applications*. Springer.
4. Robert Weinstock, *Calculus of Variations (with Applications to Physics and Engineering)*, Dover Publications, INC.
5. M. Gelfand, and S. V. Fomin, *Calculus of Variations*. Prentice–Hall, INC.

Course Learning Outcomes: Completion of the course, students should be able to

1. identify different integral equations and classify them.
2. find solution/approximate solutions, using resolvent kernels, iterated kernels etc.
3. construct Green's function for a boundary value problem.
4. solve Euler Lagrange problems and justify several aspects of the solution.
5. solve differential equation with several dependent variables.
6. apply Galerkin and Collocation methods to find approximate solutions of different problems.

Course Code	MA 726
Title of the Course	NON – LINEAR PROGRAMMING
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. introduce the concepts of convex functions and to explore convex programming problems.
2. discuss the optimality conditions for the constrained and unconstrained nonlinear programming problems.
3. study the algorithms to solve nonlinear programming.
4. learn the theory of duality in nonlinear programming.

Course Content

Problem statement – Local and Global optimality – Convex sets – Convex functions and their properties – Differentiable convex functions – Convex Programming problems.

Unconstrained optimization of functions of several variables – First-order and second- order Optimality conditions – The method of Steepest Descent – Newton's method – The conjugate gradient method.

Constrained nonlinear optimization problems – Equality and inequality constraints – Method of Lagrange multipliers – Karush-Kuhn-Tucker conditions.

Algorithms for constrained optimization - Frank-Wolfe's method – Projected Gradient methods – Beale's method – Penalty methods.

Duality in nonlinear programming – Lagrangian Dual problem – Duality Theorems – Saddle point optimality conditions – Special cases – Linear and Quadratic Programming.

Reference Books:

1. Mokhtar S. Bazaraa, Hanif D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms*, John Wiley, 2013.
2. Edwin K. P. Chong and Stanislaw H. Zak, *An Introduction to Optimization*, John Wiley,2004.
3. David G. Luenberger and Yinyu Ye, *Linear and Nonlinear Programming,* Springer, 2008.
4. Singiresu S. Rao, *Optimization: Theory and Application*, John-Wiley,2009.
5. Suresh Chandra, Jeyadeva and Aparna Mehra, *Numerical Optimization with Applications*, Narosa Publishing House, 2009.

Course Learning Outcomes: On completion of the course, the students will be able to

1. determine convex sets and convex functions and to solve convex programming problems.
2. verify the optimality conditions and to apply Steepest Descent, Newton's and conjugate gradient methods to solve unconstrained optimization problems.
3. use the method of Lagrange multipliers and Karush-Kuhn-Tucker conditions to find an optimum solution of a nonlinear programming problems.
4. solve constrained nonlinear programming using Frank-Wolf's, Projected Gradient, Beale, and Penalty methods.
5. construct the dual of a non-linear programming problem and to apply duality results to solve them.

Course Code	MA 727
Title of the Course	NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. give an understanding of numerical methods for the solution boundary value problem of ordinary differential equations, their derivation, analysis, and applicability
2. Introduce finite difference methods for partial differential equations (PDEs)
3. discuss stability, consistency, and convergence of the scheme for initial and initial boundary value problems.
4. basic idea of finite element analysis.

Course Content

Ordinary differential equations-Boundary-value problems-shooting method, finite difference methods – convergence analysis.

Parabolic equation: One dimensional parabolic equation **-** Explicit and implicit finite difference scheme Stability and convergence of difference scheme. Two dimensional parabolic equations - A.D.I. methods with error analysis.

Hyperbolic equations: First order quasi-linear equations and characteristics - Numerical integration along a characteristic - Lax-Wendroff explicit method - Second order quasi-linear hyperbolic equation - Characteristics - Solution by the method of characteristics.

Elliptic equations: Solution of Laplace and Poisson equations in a rectangular region - Finite difference in Polar coordinate Formulas for derivatives near a curved boundary when using a square mesh - Discretization error - Mixed Boundary value problems.

Finite Element Method: types of integral formulations, one- and two-dimensional elements, Galerkin formulation, application to Dirichlet and Neumann problems.

Reference Books:

1. G. Evans, J. Blackledge, P. Yardley, Numerical Methods for Partial Differential Equations, Springer Science & Business Media, 2012
2. K. W. Morton, D. F. Mayers, Numerical Solution of Partial Differential Equations: An Introduction, Cambridge University Press, 2005
3. John A. Trangenstein, Numerical Solution of Elliptic and Parabolic Partial Differential Equations, Cambridge University Press, 2013.
4. M. K. Jain, Numerical solution of Differential Equations, New Age International Publishers, 2008
5. J. N. Reddy, An Introduction to Nonlinear Finite Element Analysis: With Applications to Heat Transfer, Fluid Mechanics, and Solid Mechanics, Oxford University Press, 2015.

Course Learning Outcomes: Completion of course, student will be able to

1. Solve the boundary value problem in ordinary differential equations
2. Write finite difference numerical scheme for parabolic, elliptic, and hyperbolic equations and solve numerically.
3. discuss the convergence of numerical scheme for differential equations.
4. apply finite element method for differential equations.

Course Code	MA 729
Title of the Course	FLUID DYNAMICS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course, is to

1. understand physics involve in fluid flow problems and apply laws of conservation to construct mathematical model.
2. find mathematical solution of some viscous and inviscid flow problems

Course Content

Real Fluids and Ideal Fluids - Streamlines and Path lines; Steady and Unsteady Flows - The Velocity potential – The Vorticity vector - The Equation of continuity - Acceleration of a Fluid – Conditions at a rigid boundary - General analysis of fluid motion - -Euler's equations of motion - Bernoulli's Equation.

Discussion of a case of steady motion under conservative body forces – Some potential theorems- Some Flows Involving Axial Symmetry - Some special two- Dimensional Flows - Impulsive Motion. Some three-dimensional Flows: Introduction - Sources, Sinks and Doublets - Images in a Rigid Infinite Plane - Axi-Symmetric Flows; Stokes stream function

Two-Dimensional Flows: The stream function – The Complex Potential for Two- Dimensional, Irrotational, Incompressible Flow - complex velocity potentials for Standard Two-Dimensional Flows - The Milne-Thomson circle theorem and applications – The Theorem of Blasius

Viscous flow: Stress components in a Real fluid - relations between Cartesian components of stress - Translational Motion of Fluid Element - The Rate of Strain Quadric and Principal Stresses – Some Further properties of the Rate of Strain Quadric - Stress Analysis in Fluid Motion - Relations Between stress and rate of strain - The Navier - Stokes equations of Motion of a Viscous Fluid.

Some exact solutions of Viscous Flow - Steady Viscous Flow in Tubes of Uniform cross section - Diffusion of Vorticity - Energy Dissipation due to Viscosity - Steady Flow past a Fixed Sphere - Dimensional Analysis; Reynolds Number - Prandtl's Boundary Layer.

Reference Books:

1. F. Chorlton, Textbook of fluid dynamics, CBS Publishers & Distributors, 2005
2. J. D. Anderson, Computational Fluid Dynamics, The Basics with Applications, McGraw Hill, 2012.
3. G. K. Batchelor, An Introduction to Fluid dynamics, Cambridge University Press,2000.
4. Richard E. Meyer, Introduction to Mathematical Fluid Dynamics, Courier corporation, 2012
5. A. J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics, Springer Science & Business media, 2013.

Course Learning Outcomes: Completion of the course, student will be able to

1. understand physical concept involved in fluid motion
2. model some two- and three-dimensional flows of viscous and inviscid fluid flows.
3. find mathematical solution of some fluid flow problems and interpret results physically.

Course Code	MA 802
Title of the Course	INTRODUCTION TO FUZZY MATHEMATICS AND ITS APPLICATIONS
Prerequisite	Nil
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. fuzzy logic, fuzzy sets, properties of α-cuts, extension principles.
2. fuzzy complements, fuzzy intersection, fuzzy union.
3. fuzzy numbers, fuzzy relations, fuzzy equivalence relations.
4. classifications by fuzzy equivalence relations, c-means clustering, fuzzy c-mean clustering.
5. fuzzy decision-making methods and their applications.

Course Content

Fuzzy sets – introduction, Basic types and Basic concepts, Additional properties of □-cuts, Representation of fuzzy sets, Extension principles.

Type of operators on fuzzy sets and fuzzy complements, Fuzzy intersection and fuzzy unions, Combination of operations.

Fuzzy numbers and arithmetic operations on intervals, Arithmetic operations on fuzzy numbers, Fuzzy equations and fuzzy relations, Binary fuzzy relations and binary relations on a single set, Fuzzy equivalence relations.

Classification by equivalence relations-Crisp relations, Fuzzy relations, Cluster Analysis, Cluster Validity, c-means Clustering- Hard c-means (HCM), Fuzzy c-Means (FCM).

Fuzzy Decision making – introduction, Conversion of linguistic variables to fuzzy numbers, Individual Decision Making, Multi person decision Making, Multi criteria decision Making, Fuzzy ranking methods.

Reference Books:

1) George J. Klir, Bo Yuan, Fuzzy Sets and Fuzzy logic – Theory and Applications, first edition Pearson education India 2015.
2) H.J Zimmermann, Fuzzy sets, Decision making and expert systems, Kluwer, Bosten, 1987.
3) S.J. Chen and C. L. Hwang, Fuzzy Multiple Attributes Decision Making, Springer Verlag, Berlin Heidelberg, 1992.
4) H. J. Zimmermann, Fuzzy Set Theory and Its Application, 4th edition, Springer (SIE), 2006.
5) Timothy J. Ross, Fuzzy Logic with engineering applications, 3rd edition, Wiley, 2011.

Course Learning Outcomes: Completion of the course, student will be able to

1. Analyze the types of fuzzy sets, α-cuts and its properties and extension of functions.
2. Apply the operations (fuzzy complements, fuzzy intersections, and fuzzy unions) on fuzzy sets.
3. Create the fuzzy relations and identify the different types of fuzzy relations and their applications.
4. Apply the concepts of clustering of data (information) in engineering problems.
5. Apply the concepts of fuzzy decision-making methods in engineering and management problems.

Course Code	MA 803
Title of the Course	ADVANCED FUZZY MATHEMATICS AND ITS APPLICATIONS
Prerequisite	MA 802/ MA 720
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Operations on fuzzy numbers such as addition, subtraction, multiplication, division, Fuzzy Max and Fuzzy Min.
2. Different methods on Ranking of fuzzy numbers using Degree of Optimality.
3. Different methods on Ranking of fuzzy numbers using comparison function and scoring functions.
4. Different fuzzy decision-making methods using arithmetic operations.
5. Different fuzzy decision-making methods using maxmin composition and converting fuzzy numbers to crisp scores.

Course Content

Fuzzy Arithmetics - Fuzzy number, Addition of fuzzy numbers, Subtraction of fuzzy numbers, Multiplication of fuzzy numbers, Division of Fuzzy numbers, Fuzzy Max and Fuzzy Min, L-R Fuzzy number, Triangular (or Trapezoidal) Fuzzy Number.

Ranking Using Degree of Optimality, Watson et al.'s Approach, Baldwin and Guild's Approach, Yager's Approach, Kerre's approach, Adamo's Approach, Buckley and Chana's Approach, Mabuchi's Approach.

Ranking using comparison function, Ranking using Left and Right Scores – Jain's Approach, Chen's approach, Chen and Hwang's Approach, Ranking with Centroid Index-Yager's Centroid Index, Murakami et al.'s approach.

Fuzzy Simple Additive Weighting Methods, Bonissone's Approach, Analytic Hierarchical Process Methods- Saaty's AHP Approach, Laarhovan and Pedrycz's Approach, Buckley's Approach.

Maximin Methods – Bellman and Zadeh's Approach, Yager's Approach, A new approach to Fuzzy MADM problems-Converting Linguistic Terms to Fuzzy Numbers, Converting Fuzzy Numbers to Crisp scores, The Algorithm.

Reference Books:

1) S.J. Chen and C. L. Hwang, Fuzzy Multiple Attributes Decision Making, Springer Verlag, Berlin Heidelberg, 1992.
2) George J. Klir, Bo Yuan, Fuzzy Sets and Fuzzy logic – Theory and Applications, first edition Pearson education India 2015.
3) B. Muller, J. Reinhardt M. T. Strickland, Neural networks – An introduction, Springer verlag, Berlin Heidelberg, 1995
4) H. J Zimmermann, Fuzzy sets, Decision making and expert systems, Kluwer, Bosten, 1987.

Course Learning Outcomes: Completion of the course, student will be able to

1. Operations on fuzzy numbers such as addition, subtraction, multiplication, division, Fuzzy Max and Fuzzy Min.
2. Different methods on Ranking of fuzzy numbers using Degree of Optimality
3. Different methods on Ranking of fuzzy numbers using comparison function and scoring functions.
4. Different fuzzy decision-making methods using arithmetic operations
5. Different fuzzy decision-making methods using maxmin composition and converting fuzzy numbers to crisp scores.

Course Code	MA 804
Title of the Course	ADVANCED NUMERICAL COMPUTATIONS
Prerequisite	MA 710
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Introduce a broad range of numerical methods for solving mathematical problems that arise in Science and Engineering.
2. Adequate exposure to learn alternative methods and analyze mathematical problems to determine the suitable numerical techniques.
3. Use the concepts of interpolation, eigen value and techniques for mathematical problems arising in various fields.
4. Solve initial value and boundary value problems which have great significance in engineering practice using ordinary and partial differential equations.
5. This course will provide basic fundamental knowledge and experience of work with numerical methods necessary for applied mathematicians and applied scientists.

Course Content

Unit – I: The Solutions of Numerical algebraic and Transcendental equations - Solution of system of linear algebraic equations

Unit – II: Finite differences - Lagrange, Hermite and Spline interpolation - Numerical differentiation and integration

Unit - III: Numerical solutions of ODEs. Basic theory and application of Single step and multistep methods. General theory of homogeneous and non-homogeneous linear ODEs - Variation of parameters. Eigen value problems - Initial value problems - two-point boundary value problems. Stability analysis- zero stability, absolute stability, relative stability and interval of stability.

Unit - IV: Classification of second order PDEs - General solution of higher order PDEs with constant coefficients - Method of separation of variables for Laplace, Heat and Wave equations. Difference Methods for Parabolic, Hyperbolic and Elliptic equations. Convergence, consistency and stability analysis.

Unit - V: The Finite Element Method: Functional- Base Function Methods of Approximation- The Rayleigh –Ritz Method –The Galerkin Method, Application to Two-dimensional problems. Finite element Method for one and two-dimensional problems.

Reference Books:

1) M. K. Jain, Numerical solution of differential equations, New age international publishers, New Delhi, 2008
2) K. Shankara Rao, Numerical Methods for Scientists and Engineers, Prentice Hall of India, 2001.
3) K. Atkinson, and Weimin Han, Elementary Numerical Analysis, 3rd edition, John Wiley & Sons, Canada, 1978.
4) S.S. Sastry, Introductory Methods of Numerical Analysis, 4th edition, PHI Learning Pvt. Lmt., New Delhi, 2009.

Course Learning Outcomes: Completion of the course, student will be able to

1. Understand the concept of getting the Numerical solutions of ordinary differential equations.
2. Identify and interpret the fundamental concepts of polynomials and roots of equations, finite differences, eigenvalues and eigenvectors and corresponding algorithms and computer programs.
3. Apply the knowledge and skills of numerical methods to solve algebraic and transcendental equations, solution of ODE using spline interpolation, eigenvalue problems numerically using computer programs.
4. Analyze the physical problem to establish mathematical model and use appropriate method to solve and optimize the solution of roots of equations in engineering practice, interpolating the polynomial, boundary value problems of ODE and PDE, eigenvalue problems numerically using computer programs.
5. Distinguish the overall mathematical knowledge gained to demonstrate and analyze the problems of finding the roots of equations, interpolation, differential equations, eigenvalue problems arising in real-life situations.

Course Code	MA 805
Title of the Course	ADVANCED NUMERICAL ANALYSIS ON DIFFERENTIAL EQUATIONS
Prerequisite	MA 707, MA 710
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Introduce a broad range of numerical methods for solving mathematical problems that arise in Science and Engineering.
2. Provide a basic understanding of the derivation, analysis, and use of these numerical methods, along with a rudimentary understanding of finite precision arithmetic and the conditioning and stability of the various problems and methods.
3. Solve initial value and boundary value problems which have great significance in engineering practice using ordinary and partial differential equations.
4. Provide basic fundamental knowledge and experience of work with numerical methods necessary for applied mathematicians and applied scientists.

Course Content

Topology of Real numbers, Limit & Continuity of functions: Intervals-Neighborhood of a point-Deleted neighborhood of a point-Theorems on neighborhood-Open set-Theorems on open set-Interior point of a set-Theorems of interior on a set-closed set-Theorems on closed sets-Limit point of a set-Isolated point-Derived set-Adherent point-Closure of a set-Perfect set-Dense set-Theorems on limit points-compact set-Theorem-Open cover-Heine-Beine-Borel property-Limits at infinite-Infinite limits-Limit of a function at a point-Uniqueness-Algebra of limits-Continuity definitions-Algebra of continuous functions-Boundedness of continuous functions-Intermediate value theorem-Monotonic functions-Uniform continuity.

Ordinary Differential Equations-Initial Value Problem: Introduction-Difference Equations-Numerical Methods-Single step methods-Stability analysis of single step methods-Multistep methods-Predictor-Corrector methods-Stability analysis of Multistep methods-Stiff system.

Ordinary Differential Equations-Boundary Value Problem: Introduction-Initial value problem method (shooting method)-Linear second order differential Equations-Nonlinear second order differential Equations-Local Truncation Error-Derivative Boundary Conditions-Solution of tridiagonal system-Finite element methods.

Partial Differential Equation: Introduction-Difference methods for parabolic PDEs-Difference methods for Hyperbolic PDEs-Difference methods for Elliptic PDEs-Finite element methods.

Error Estimation of ODE and PDE: Error estimates of the Ordinary differential equations and Partial differential equations.

Reference Books:

1) M. K. Jain, Numerical solution of differential equations, New age international publishers, New Delhi, 2008.
2) N. P. Bali, Real Analysis, Golden Maths Series, Firewal Media publishers, New Delhi, 2011.
3) M. D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Ltd, New Delhi, 2010.
4) M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical methods for scientific and engineering computation, New age international publisher, New Delhi, 2010.

Course Learning Outcomes: Completion of the course, student will be able to

1. Understand the concept of getting the Numerical solutions of ordinary differential equations.
2. Classify the fundamental principles of partial differential equations (PDEs) and to solve the hyperbolic, parabolic and elliptic equations.
3. Apply theoretical concepts in topology to understand the real-world applications.
4. Apply specific methodologies, techniques and resources to conduct research and produce innovative results in the area of specialization.
5. Demonstrate ability to think critically by determining and using appropriate techniques for solving a variety of differential equations.

Course Code	MA 806
Title of the Course	MATRIX THEORY AND STOCHASTIC PROGRAMMING
Prerequisite	MA
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is

1. To introduce probability theory.
2. To understand the random vectors and distributions.
3. To understand matrix theory.
4. To understand vector spaces and inner product spaces.
5. To analyze the discrete moment problems using Linear programming

Course Content

PROBABILITY: Probability space – Events – Sample space.

RANDOM VARIABLES: Discrete and continuous random variables – Distributions – Moments.

MATRICES: Singular value decomposition - Canonical Forms – Generalized Inverse.

LINEAR ALGEBRA: Vector spaces – Inner product spaces.

LINEAR PROGRAMMING: Primal and Dual Simplex methods – Integer programming – Simulation – Discrete Moment problem.

Reference Books:

1) Introduction to Mathematical Statistics, Hogg, Mckean, Craig, Pearson Education.
2) Topics in Algebra, I. N. Herstein, John Wiley and sons – India edition.
3) Matrix Theory, James M. Ortega, Plenum Press, New York.
4) Operations Research: An Introduction, Hamdy A. Taha , Prentice Hall of India Private limited, 2004
5) Stochastic Programming, Andras Prekopa, Kluwer Academic publishers.

Course Learning Outcomes: Completion of the course, student will be able to

1. Understand probability theory in depth.
2. Understand the random vectors and distributions.
3. Understand matrix theory.
4. Get insight into vector spaces and inner product spaces.
5. Analyze the discrete moment problems using linear programming.

Course Code	MA 850
Title of the Course	ADVANCED NUMERICAL ANALYSIS FOR SINGULARLY PERTURBED DIFFERENTIAL EQUATION
Prerequisite	MA 710
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. To understand the Fitted operator method for Singularly Perturbed Differential Equations.
2. Apply various methods for Boundary Value Problem
3. To discuss about special numerical methods for SPP.
4. To Know about FEM for SPP, With various boundary conditions.
5. To know the numerical schemes for more than one dimension

Course Content

Simple examples of singularly Perturbation Problem. Uniformly distributed numerical method for problems with initial and boundary layers: initial value problems- some uniformly convergent difference schemes, constant fitting factors, optimal error estimates.

Boundary value problems- constant fitting factors for a self-adjoint problem, non-self-adjoint problem, self-adjoint problem in conservation form, non-self-adjoint problem in conservation form, problems with mixed boundary conditions, fitted verses standard method, experimental determination of order or uniform convergence.

Fitted operator method- fitted mesh method, cubic spline method, finite element method, variable mesh method, shooting method, collocation method, booster method, boundary value technique, initial value technique, Schwartz method and convergence of the above methods- reaction- diffusion, convection-diffusion, reaction- convection-diffusion type problem in one dimension.

Simple fitted mesh methods in one dimension, convergence of fitted mesh finite difference methods for linear convection-diffusion problems in one-dimension, linear convection-diffusion problems in two dimensions and their numerical solutions, fitted numerical methods for problems with initial and parabolic boundary layers.

Finite element method and finite volume method for Singularly Perturbed ode and singularly perturbed pde of linear problems.

Reference Books:

1) M. K. Jain, S. R. K. Iyengar, R. K. Jain, Numerical Methods for scientific and engineering computation, New Age international publishers, 5th edition, 2007.
2) J. J. H. Miller, E. O'Riordan, G. I. Shishikin, Fitted numerical methods for singular perturbation problems, world scientific. (2000)
3) C. M. Bender, S. A. Orszag, advanced Mathematical methods for Scientists and engineers, Springer, New York, 1999
4) Lauurence C. Evans, Partial differential equations, Graduate studies in Mathematics, Vol. 19, American Mathematical Society, Providence, 1998
5) Robert C Mc Owen, Partial Differential Equations- Methods and Application, Pearson Education Inc., Indian reprint 2004

Course Learning Outcomes: Completion of the course, student will be able to

1. Can Apply Fitted operator method for SPP.
2. Able to find the solution of SPP by FEM.
3. Can apply special schemes to spp like Schwarz and IVT,etc.
4. Obtain the knowledge in more than one dimensional spp also.
5. Able to solve problems with various boundary conditions.

Course Code	MA 852
Title of the Course	BIORHEOLOGY
Prerequisite	MA 729
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. introduce fluids and their properties.
2. append knowledge on blood and properties.
3. learn rheological models of blood vessels.
4. know about the terms like stenosis, thrombosis, and Atherosclerosis.
5. study pulsatile flow and Permeability.

Course Content

Fluid – Newtonian and Non-Newtonian flow – Laminar flow in a cylindrical tube – Turbulent flow – Suspensions – Hookean Solid – Viscoelasticity – Strain energy density function – Thixotrophy.

Blood – Aggregation and Sedimentation of red cells – Non-Newtonian viscosity - Thixotrophy and viscoelasticity – Factors affecting blood viscosity - Casson fluid – Plasma layer – Radial migration – Flow of red cell suspension in small tubes – Fahraeus effect ,Fahraeus-Lindqvist effect – wall surface effect - Copley- Scott Blair phenomenon – Disturbed flows of red cell suspensions – Viscosity of blood clots – Blood rheology at near-zero gravity .

Blood vessel walls – Forces in blood vessel walls – General theory of circumferential tension – Stress distribution in blood vessel walls - Incremental theory of blood vessel walls – Nonlinear theory of elastic deformation – Tethering effect on the stresses in blood vessels – Some rheological models of blood vessels.

Pulse – Theoretical studies of pulse waves – oscillatory flow in a rigid tube – Wave propagation in elastic tubes – pressure - flow relationship – Pulsatile flow in microvessels

Flow in a locally constricted tube – Post-stenotic dilatation- Flow at branching sites – Thrombosis – Atherosclerosis – Protein uptake by arterial wall – Permeability and pathways of macromolecules – Physical theory of vascular permeability to proteins – Stresses in the arterial wall as a cause of permeability.

Reference Books:

1) Syoten Oka.,Cardiovascular Hemorheology,Cambridge University press,London,1981.
2) Y.C.Fung., Biomechanics: Its foundations and objectives, Prentice-Hall, 1973.
3) Arthur S.Lodge, Michael Renardy and John A.Nohel., Viscoelasticity and Rheology, Academic press, Inc. Newyork, 1985.
4) J.C. Misra, Biomathematics: Modeling and Simulation, World Scientific Publishing
5) Company, 2006.

Course Learning Outcomes: Completion of the course, student will be able to

1. Apply the concepts of fluidity for their problems.
2. Develop mathematical models.
3. Introduce the nature of pulsatality and permeability.
4. Identify the stenosis of different forms.
5. Contribute useful results to the society.

Course Code	MA 854
Title of the Course	INTRODUCTION TO SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS
Prerequisite	MA 707, MA 708
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is

1. To find the asymptotic solution of the singular perturbation problems.
2. To know the boundary layer behavior of the singular perturbation problems.
3. To discuss the existence and stability of the solutions of SPP.
4. To understand WKB method for SPP.

Course Content

Mathematical Preliminaries: Little-oh, Big-oh, asymptotically equal to or behaves like notation, Asymptotic sequence and expansions, Convergent series versus divergent series, Asymptotic expansions with parameter, uniformity or breakdown, overlap regions, the matching principle, matching with logarithmic terms, composite expansion.

Introductory application: Roots of equations, integration of functions represented by asymptotic expansions, ordinary differential equations: regular problems, simple singular problems, scaling of differential equations, equations which exhibit a boundary layer behavior, where is the boundary layer?

Motivation for the study of singular perturbation problem (SPP): Asymptotic expansion and approximation, asymptotic solution of algebraic and transcendental equations, regular and singular perturbations for first and second-order ordinary differential equations, physical examples.

Two-point boundary-value problems: Boundary layer- exponential and cusp layers, marched asymptotic expansions, composite asymptotic expansions, WKB method-conditions for validity of the WKB approximation, patched asymptotic approximations, WKB solution of inhomogeneous ordinary differential equations.

Boundary layers and transition layers. The method of multiple scales: Nearly linear oscillations, nonlinear oscillators, applications to classical ordinary differential equations, WKB method for slowly varying oscillations, turning point problem, applications to partial equations, limitation on the use of the method of multiple scales, boundary layer problems. Some physical applications of singular perturbation problems.

Reference Books:

1) C.M. Bender, S.A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, Springer, New York, 1999.
2) R.E. O'Malley, Singular Perturbation Methods for ordinary differential equations, Springer-Verlag, New York, 1991
3) Robert C, Mc Owen, Partial Differential Equations- Methods and Applications, Pearson Education Inc, Indian Reprint 2004
4) E.A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall India, 1995
5) A.H. Nayfeh, Introduction to perturbation Techniques, John Willey & Sons, New York, 1981

Course Learning Outcomes: Completion of the course, student will be able to

1. Able to derive the asymptotic approximations singular perturbation problems.
2. Can deduct the boundary layer behavior of SPP.
3. Able to understand the existence and stability of the given problem.
4. Apply and analyze the WKB method.

Course Code	MA 855
Title of the Course	ADVANCED COMPLEX ANALYSIS
Prerequisite	MA 704
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:
The course presents an introduction to some topics of contemporary complex analysis, in particular spaces of analytic functions, univalent functions etc. The purpose is to prepare the student for an independent work in these topics and specially to use the methods of complex analysis in other areas of mathematics. The students should be able to participate in scientific discussions and conduct research on contemporary and classical complex analysis and its applications.

Course Content
Unit I. Convex functions, Hadamard three lines and circles theorems, Phragmen-Lindelof theorem.

Unit II. The space of continuous functions C(G,**Ω**), normal family, spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem

Unit III. Infinite products, Weiersirass' factorization theorem, factorization of the sine function. The gamma functions, The Riemann zeta function, Runge's theorem, simply connected regions, Mittag-Leffler's theorem

Unit IV. Harmonic functions, maximum and minimum principles, harmonic functions on a disk, Harnack's theorem, Dirichlet problem for disk, Green's function.

Unit V. Entire functions. Jensen's formula, Bloch's theorem, Picard theorems, Schottky's theorem.

Reference Books:
1) Conway John. *Functions of One Complex Variables.* 2nd ed, Narosa, New Delhi. 2002.
2) Ahlfors Lars. *Complex Analysis*. McGraw Hill Co., New York. 1988.
3) Hahn Liang-Shin and Epstein Bernard. *Classical Complex Analysis*. Jones and Bartlett India, New Delhi. 2011.
4) Rudin Walter. *Real and Complex Analysis*. McGraw-Hill. 1987.
5) Ullrich David. *Complex Made Simple*. American Math. Soc., Washington DC. 2008.

Course Learning Outcomes: Completion of the course, student will be able to
1. Explain the fundamental concepts of complex analysis and their role in modern mathematics and applied contexts.
2. Demonstrate accurate and efficient use of complex analysis techniques.
3. Demonstrate capacity for mathematical reasoning through analyzing, proving, and explaining concepts from complex analysis.
4. Apply problem-solving using complex analysis techniques applied to diverse situations in physics, engineering, and other mathematical contexts.

Course Code	MA 856
Title of the Course	GEOMETRIC FUNCTION THEORY
Prerequisite	MA 704
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:
The course presents an introduction to univalent function theory and various special classes. The purpose is to analyze the interplay of geometry and analysis. The students should be able to conduct research on contemporary and classical complex analysis and its applications.

Course Content
Unit I. Univalent functions, area theorem, growth and distortion theorems, coefficient estimates for univalent functions, special classes of univalent functions (starlike, convex, close-to-convex, spiral, typically real functions).
Unit II. Lowner's theory and its applications. Caratheodory convergence theorem, density of slit mappings, Lowner's differential equation, third coefficients, radius of starlikeness, rotation theorem, Robertson's conjecture, outline of de Banges proof of Bieberbach conjecture.
Unit III. Generalization of the area theorem, Faber polynomials, polynomial area theorem, Grunsky inequalities, inequalities of Goluzin and Lebedev, exponentiation of the Grunsky inequalities, Logarithmic coefficients.
Unit IV. Subordination. Coefficient inequalities, sharpened form of Schwarz lemma, majorization, univalent subordinate functions, Baernstein's theorem on integral means, convolution and sections of univalent functions.
Unit V. Theory of differential subordination. Jack-Miller-Mocanu lemma, admissibility conditions, first and second order differential subordination and applications.

Reference Books:

1) Conway John. *Functions of One Complex Variables*. vol. 2, Springer, New York. 1973.
2) Duren Peter. *Univalent Functions*. Springer, New York. 1983.
3) Goodman Adolph W. *Univalent Functions I & II.* Mariner, Florida. 1983.
4) Pommerenke Christian and Jensen Gerd. *Univalent Functions.* Van den Hoek and Ruprecht, Göttingen. 1975.
5) Rosenblum Marvin and Rovnyak James. *Topics in Hardy Classes and Univalent Functions.* Birkhauser Verlag. 1994
6) Hallenbeck David J. and MacGregor Thomas H. *Linear Problems and Convexity Techniques in Geometric Function Theory.* Pitman Adv. Publ. Program, Boston-London-Melbourne. 1984.
7) Graham Ian and Kohr Gabriela. *Geometric Function Theory in One and Higher Dimensions*. Marcel Dekker, New York. 2003.

Course Learning Outcomes: Completion of the course, student will be able to
1. Explain the fundamental concepts of univalent functions and various special classes like convex, starllike, close to convex etc.
2. Demonstrate Lowner's theory and its role in deriving Bieberbach conjecture.
3. Explain Faber polynomials and how it helps in generalizing area theorem and various other inequalities.
4. Understand the subordination theory and its relationship with geometric function theory.

Course Code	MA 857
Title of the Course	APPROXIMATION THEORY
Prerequisite	MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:

1. The course presents an introduction to approximation of continuous functions by polynomials and the best approximation by subsets of normed linear spaces or inner product spaces.
2. It introduces upper, lower semi continuity of the metric projection maps and the continuous selections of the set valued maps.
3. This course prepares the students to conduct research on approximation theory.

Course Content

Approximation by Algebraic Polynomials - Approximation of Periodic Functions - Convergence of Bernstein Polynomials - Korovkin's Theorem - Stone-Weierstrauss theorem.

Chebyshev's Alternation theorem - General linear families - Haar system and its characterizations - uniqueness of polynomials of best approximation - strong unicity theorem - Harr's unicity theorem. orthogonal polynomials - Legendre and Chebyshev polynomials.

Strictly convex and uniformly convex Banach spaces – Best approximation in inner product spaces – Best approximation from closed, convex subsets - continuity of metric projection.

Best approximation by subspaces of Banach spaces - Duality formula - proximinal sets - Chebyshev sets - proximinality of weak* closed subspaces.

Upper semi continuity and Lower semi continuity of the set-valued maps - Continuous selections and Lipschitz Continuity of Metric Projections.

Reference Books:

1) Ward Cheney, Will Light: *A Course in Approximation theory*, Graduate Studies in Mathematics, American Mathematical Society, 2013.
2) Frank Deutsch: *Best Approximation in Inner Product Spaces*, Springer, 2001.
3) Serge Lang: *Real and Functional Analysis*, Springer, 1993.
4) Ivan Singer*: Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces*, Springer-Verlag, 1970.
5) H. N. Mhaskar, D. V. Pai: *Fundamentals of Approximation Theory*, Narosa, 2000.

Course Learning Outcomes: Completion of the course, student will be able to

1. Explain the approximation of continuous functions by polynomials.
2. Know some applications of Chebyshev alternation theorem.
3. Explain the best approximation from closed convex subsets of inner product spaces and continuity of metric projection.
4. Understand the best approximation by subspaces of Banach spaces.
5. Explain the semi continuity properties of set-valued maps and continuous selection of metric projection maps.

Course Code	MA 858
Title of the Course	CONVEX ANALYSIS
Prerequisite	MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:

1. The course presents an introduction to convex functions, extremal structure of convex sets.
2. The students will understand the extreme points and differentiability of convex functions. In particular, derivatives of norm functions.
3. This course prepares the students to conduct research on classical analysis and its applications.

Course Content

Convex sets - convex hull and affine hull – convex subsets of R^n – Caratheodory's theorem - Helley's theorem - Convex hull of a compact set is compact.

Convex functions on Rn - one sided derivatives - continuity and differentiability - Directional derivatives and subgradients - Differentiable convex functions- Inequalities.

Extremal subsets and extreme points of convex sets- Extreme points of unit ball of Banach spaces - Krein Milman theorem – Milman converse.

Differentiability of convex functions defined on Banach spaces - sublinear functionals and one-sided derivatives - Gateaux and Frechet derivatives - Mazur's theorem.

Derivatives of the norm function and the duality maps- Smulian's characterization of the differentiability of the norm function.

Reference Books:

1) Jan Van Tiel: *Convex Analysis*, John Wiley, 1984.
2) R. R. Phelps: *Convex functions, Monotone operators and Differentiability* Springer Lecture Notes, 1993.
3) Marian Fabian, Petr Habala, Petr Hajek, V. M. Santalucia, J.Pelant and V. Zizler: *Functional Analysis and Infinite-Dimensional Geometry*, CMS Books in Mathematics, 2001.
4) R. Tyrrell Rockafeller, *Convex Analysis*, Princeton University Press, 1972.

Course Learning Outcomes: Completion of the course, student will be able to

1. Explain the convex sets and convex functions.
2. Understand the directional derivatives and sub gradients of convex functions.
3. Explain the extreme points of convex sets and know some applications of Krein-Milman theorem.
4. Find Gateaux and Fréchet derivatives of convex functions defined on Banach spaces.
5. Explain the derivatives of norm functions and duality maps.

Course Code	MA 859
Title of the Course	MATRIX ANALYSIS
Prerequisite	MA 703
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is

1. To learn partitioned matrices and Jordan canonical and Singular value decompositions of matrices.
2. To investigate Hadamard and Kronecker products and related matrix inequalities.
3. To study characterizations and properties of positive semidefinite matrices.
4. To discuss nonnegative matrices and Perron-Frobenius Theorem.
5. To introduce different classes of matrices and study their relationship with complementarity problems and stability analysis.

Course Content

Review of basic matrix theory – Determinant and inverse of partitioned matrices – Annihilating polynomial of matrices – Minimal polynomial and diagonalization of matrices – Jordan canonical forms – Spectral theorem – Singular value decomposition.

Positive semidefinite matrices – Characterization and properties – Simultaneous diagonalization – Schur Complement – Fischer and Hadamard inequalities – Kronecker and Hadamard products.

Nonnegative matrices – Irreducible matrices – Perron-Frobenius Theorem - Matrices with positive principal minors (P- matrices) – Sign reversal properties – Global uniqueness of linear complementarity problems.

Matrices with nonnegative principal minors (P0 matrices) – Sufficient matrices – Nondegenerate matrices – Copositive matrices – Semimonotone matrices – Completely Q-matrices.

Z-matrices and Least-element Theory – M-matrices – Characterizations – Positivity of principal minors – Positive stability – Inverse-positivity and splittings – Stieltjes matrices.

Reference Books:

1) Fuzhen Zhang, “Matrix Theory - Basic Results and Techniques”, Springer, 1999.
2) Roger A. Horn and Charles R. Johnson, “Matrix Analysis”, Cambridge University Press, 1990.
3) Abraham Berman and Robert J. Plemmons, “Nonnegative Matrices in the Mathematical Sciences”, SIAM, 1994.
4) Richard W. Cottle, Jong-Shi Pang, Richard E. Stone, “The Linear Complementarity Problem”, SIAM, 2009.

Course Learning Outcomes: Completion of the course, student will be able to

1. Determine the determinant and inverse of partitioned matrices
2. Construct Jordan canonical form and singular value decomposition of a matrix.
3. Derive matrix inequalities using Schur complements, Hadamard and Kronecker products.
4. Identify positive semidefinite matrices through eigenvalues, minors, and determinants and to compute Perron vector.
5. Categorize matrix classes and use them in stability analysis and complementarity problems.

Course Code	MA 860
Title of the Course	OPTIMIZATION TECHNIQUES
Prerequisite	MA 701, MA 703
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is

1. To introduce the notion of convex sets and convex functions, and to discuss Extreme points and extreme directions of Polyhedral sets.
2. To study first-order and second-order optimality conditions for unconstrained optimization problems.
3. To learn Fitz John and Kuhn-Tucker conditions for constrained optimization problems.
4. To discuss methods to solve unconstrained and constrained optimization problems.
5. To introduce the Lagrangian dual problem and to study properties of the dual function.

Course Content

Convex sets – Convex Hulls – Caratheodory Theorem – Separation Theorems and Farkas' Lemma – Convex cones – Polyhedral sets – Extreme points – Linear Programming.

Convex Optimization – Convex functions – Epigraph – Directional Derivative – Subgradients – Differentiable convex functions – Hessian matrix – Local and global optimality.

Unconstrained optimization – Optimality conditions – First-order and second-order conditions –The method of Steepest Descent – Newton's method – The conjugate gradient method.

Constrained optimization – Problems with equality and inequality constraints – Method of Lagrange multipliers – Fitz John conditions – Karush-Kuhn-Tucker conditions – Frank-Wolfe's method – Projected Gradient methods – Penalty methods.

Lagrangian duality – Geometric Interpretation – Duality Theorems – Saddle point optimality – Duality for linear and quadratic optimization problems.

Reference Books:

1) Mokhtar S. Bazaraa, Hanif D. Sherali and C. M. Shetty, "Nonlinear Programming Theory and Algorithms", John Wiley, 2013.
2) E. K. P. Chong and Stanislaw H. Zak, "An Introduction to Optimization", John Wiley, 2004.
3) Niclas Andreasson, Anton Evgrafov and Michael Patriksson, "An Introduction to Continuous Optimization", Overseas Press, 2006.
4) Suresh Chandra, Jeyadeva and Aparna Mehra, "Numerical Optimization with Applications", Narosa, 2009.
5) Stephen Boyd and Lieven Vandenberghe, "Convex Optimization", Cambridge University Press, 2009

Course Learning Outcomes: Completion of the course, student will be able

1. To analyze convex sets and convex functions and to compute subgradients of a convex function.
2. To verify optimality conditions of an unconstrained optimization problem and to solve it using Steepest Descent, Newton's and conjugate gradient methods.
3. To find an Optimum solution to a constrained optimization problem using Karush-Kuhn-Tucker and Fritz John conditions.
4. To determine an optimal solution of constrained optimization problem using Simplex, Frank-Wolfe's, Projected Gradient and Penalty methods.
5. To write Lagrangian dual problem to a non-linear programming (NLP) and to apply duality results to solve NLP

Course Code	MA 861
Title of the Course	HARDY – HILBERT SPACES
Prerequisite	MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:
This course introduces various concepts related to Hardy-Hilbert Space. It includes a study of shift operators, invariant subspaces, and inner and outer functions. It also deals with Toeplitz, Hankel and Composition operators.

Course Content

The Hardy Hilbert Space: Basic Definitions and properties. The unilateral shift and factorization of Spectral structure. functions: Shift operators, Invariant and reducing subspaces.

Inner and outer factorization, Blaschke factors, singular inner functions, outer functions. Toeplitz operators: Basic properties of Toeplitz operators, spectral structure.

Hankel operators: Bounded Hankel operators, Hankel operators of finite rank, Compact Hankel operators, self adjointness and normality of Hankel operators. Relation between Hankel and Toeplitz operators.

Composition Operators: Fundamental Properties of Composition Operators, Invertibility of Composition Operators, Eigenvalues and Eigenvectors, Composition Operators Induced by Disk Automorphisms.

Reference Books:

1. R.A. Martinez-Avedano and P. Rosenthal, An Introduction to the Hardy-Hilbert Space, Graduate Texts in Mathematics 237, Springer, 2007.

2. R.G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998

3. N.K. Nikolskii, Operators, Functions and Systems: An Easy Reading, Volume 1, Mathematical Surveys and Monographs 92, American Mathematical Society, 2002

Course Learning Outcomes: Completion of the course, student will be able to
1. identity Hardy-Hilbert spaces in several equivalent forms and know various properties of shift operators.
2. prove theorems about Hankel operators and Toeplitz operators
3. prove theorems about Composition operators.

Course Code	MA 862
Title of the Course	ASYMPTOTICS AND PERTURBATION METHODS
Prerequisite	MA 707
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives:
The overarching goal of this course is to introduce the fundamentals of asymptotic analysis and perturbation methods for solving problems arising in the study of differential equations and integrals

Course Content
Introduction to asymptotic approximations: Definitions; Convergence; Asymptotic Ness; Parametric expansions. Asymptotic analysis of integrals: Solutions to algebraic equations: Regular and singular perturbations.

Regular perturbation problems in ODEs and PDEs: Initial value problems; Boundary perturbations. Introduction to singular perturbation of ODEs. Poincare-Lindstedt method.

Boundary layer theory, WKB Theory, Multiple-scale analysis, Introduction to singular perturbation of PDEs**,** Engineering applications: At least one example each from fluid mechanics, solid mechanics, and vibrations.

Reference Books:

1. Hinch, E. J., 1991. Perturbation Methods, Cambridge U. Press: Cambridge, U.K.
2. Murdock, J. A., 1987. Perturbations: Theory and Methods, SIAM.
3. Van Dyke, M., 1975. Perturbation Methods in Fluid Mechanics. Parabolic Press.
4. Kevorkian, J., and J. D. Cole, 1981. Perturbation Methods in Applied Mathematics. Springer
5. Holmes, M. H. 2013. Introduction to Perturbation Methods. Springer.
6. D. Wilcox (1995) Perturbation methods in the computer age. DWC Industries Inc.
7. Bender & S. Orszag (2010) Advanced mathematical methods for scientists and engineers. Springer

Course Learning Outcomes: Completion of course student will be able to
1. Explain basic concepts of perturbation techniques, such as order relationships, asymptotic sequences, asymptotic expansions, and convergence issues.
2. Propose a solution method for regular perturbation problems
3. Explain the difference between a regular and a singular perturbation problem
4. Analyze a singular problem by means of a balancing method
5. Determine inner and outer solutions for singular perturbation problems by means of boundary-layer theory and the composite form
6. Use WKB methods to solve linear ordinary differential equations subjected to different length or time scales
7. Perform a multiple-scale analysis on linear and non-linear problems Apply perturbation methods to partial-differential problems

Course Code	MA 863
Title of the Course	SOBOLEV SPACES AND ITS APPLICATIONS
Prerequisite	MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to introduce

1. The knowledge of functional analysis in real life problems
2. Existence and Uniqueness of regular solutions.

Course Content

Introduction: Elements of operator theory and Hilbert spaces.

Distribution Theory: Test functions and distributions, Convolution of Distributions, Tempered Distributions.

Sobolev Spaces: Definition and basic properties, Extension Theorems, Imbedding theorems, Compactness theorems, Trace theory.

Weak Solutions of Elliptic boundary value problems: Examples of Elliptic BVPs, Existence and Regularity of weak solutions, Maximum principle, Eigenvalue problems.

Reference Books:

1. S. Kesavan, *Topics in Functional Analysis and Applications*, New Age International Publishers, 2015.
2. H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2011.
3. M. Renardy and R. C. Rogers, *An Introduction to Partial Differential Equations*, Springer, 2004.

Course Learning Outcomes: Completion of the course, students should be able to

1. identify Sobolev spaces
2. apply functional analysis to study real life problems (existence, uniqueness of regular solutions).

Course Code	MA 864
Title of the Course	GRAPHS AND MATRICES
Prerequisite	MA 703
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. To study block matrices, spectral theorem, eigenvalue inequalities and Perron-Frobenius Theorem, and to present graph operations and products.
2. Using the ideas and techniques of matrix theory to study graph theoretic properties through its associated graph matrices.
3. To analyze the concepts of the Incidence matrix and the Adjacency matrix of a graph
4. To introduce the notion of the Laplacian matrix of a graph and to discuss its properties.
5. To investigate the Distance matrix of a graph and to study its determinant, inertia, and inverse.

Course Content

Determinant and inverse of partitioned matrices - Schur complement of a block matrix- Spectral theorem - Rayleigh quotient for Hermitian matrix - Cauchy's Interlace Theorem for Eigenvalues - Perron-Frobenius Theorem.

Basic notions in Graph theory - Euler's theorem - Line graphs - Operations on graphs: complement, union and join of graphs - Graph products: Cartesian product and composition of graphs - Vertex/Edge Independent Sets.

Incidence Matrix - Properties of Incidence Matrix - Adjacency Matrix - The Adjacency spectrum - Sachs' theorem - Determinant of the Adjacency Matrix of simple graphs.

Laplacian Matrix - Properties of Laplacian Matrix - Matrix-tree theorem - Laplacian eigenvalues of trees.

Distance Matrix - Determinant and Inverse of Distance Matrix of trees - Relation between Distance and Incidence Matrix of trees - Inertia of distance matrix of a tree.

Reference Books:

1) R. Balakrishnan and K. Ranganathan, A textbook of graph theory, 2nd Ed, Universitext, Springer, New York, 2012.
2) R. B. Bapat, Graphs and matrices, Universitext, Springer, London, 2010.
3) A. E. Brouwer and W. H. Haemers, Spectra of graphs, Universitext, Springer, New York, 2012.
4) D. Cvetkovic, P. Rowlinson and S. Simic, An introduction to the theory of graph spectra, London Mathematical Society Student Texts, 75, Cambridge University Press, Cambridge, 2010.
5) F. Harary, Graph theory, Addison-Wesley Publishing Co., Reading, MA,1969.
6) R. A. Horn and C. R. Johnson, Matrix analysis, 2nd Ed, Cambridge University Press, 2013.

Course Learning Outcomes: Completion of the course, student will be able to

1) compute determinant and inverse of block matrices using Schur complement and construct different classes of graphs using various graph operations
2) apply the Perron-Frobenius theorem and the Rayleigh quotient to several graph matrices to obtain bounds for graph theoretic parameters
3) give the rank and the column space of the Incidence matrix of a graph and derive eigenvalue inequalities between graphs and its induced subgraphs using interlacing theorem
4) find coefficients of the characteristic polynomial of the Adjacency matrix and relate cofactors of the Laplacian matrix of a graph with its spanning trees
5) determine inertia, determinant, and inverse formula for Distance matrix of trees.

Course Code	MA 865
Title of the Course	APPLIED FUNCTIONAL ANALYSIS
Prerequisite	MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Application of functional analysis in real life problems
2. Analyze different Odes and PDEs
3. Existence and Uniqueness of regular solutions.
4. Study the behavior and nature of differential equations
5. Do research in different fields of applied mathematics

Course Content

Test functions and distributions, Convolution of Distributions, Tempered Distributions.

Definition and basic properties, Approximations by smooth functions; Extension Theorems, Imbedding theorems, Compactness theorems, Trace theory.

Examples of Elliptic BVPs, Existence and Regularity of weak solutions, Maximum principle, Eigenvalue problems.

Applications to Real Life problems: Heat equation; Wave equation.

Reference Books:

1) S. Kesavan, Topics in Functional Analysis and Applications, New Age International Publishers, 2015
2) H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011
3) M. Renardy and R. C. Rogers, An Introduction to Partial Differential Equations, Springer, 2004.
4) Giovanni Leoni. A First Course in Sobolev Spaces (Graduate Studies in Mathematics). Springer
5) Sandro Salsa. Partial Differential Equations in Action: From Modelling to Theory. Springer

Course Learning Outcomes: Completion of the course, student will be able to

1. Identify Sobolev spaces
2. Apply functional analysis to study real life problems
3. Study existence, uniqueness of regular solutions
4. Analyze the solutions of ODEs and PDEs
5. Do research in different fields of applied mathematics

Course Code	MA 866
Title of the Course	PARTICULATE PROCESSES: THEORY AND MODELLING
Prerequisite	MA 710, MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Population balance problems in real life: Discrete and continuous models
2. Existence and Uniqueness of regular solutions.
3. Numerical approximations using sectional methods and methods of moments
4. Stability, consistence, and convergence of solutions
5. C_0 – Semigroup approach for analytical treatment of the solutions

Course Content

Introduction of different mathematical models representing particulate processes: Discrete and continuous.

Mathematical study on the behavior of the solutions: Existence, Uniqueness, Equilibria via Lyapunov functional, Gelling, Self-similar and asymptomatic behaviors.

Numerical discretization: Sectional methods, Method of moments, Semi-analytical methods.

From the discrete to continuous models: Stability and convergence.

C_0 –Semigroup approach for analytical treatment of the solutions.

Reference Books:

1) Jacek Banasiak and Wilson Lamb, Phillipe laurençot. Analytic methods for coagulation-fragmentation models. Volume I. Boca Raton, FL: CRC Press
2) Jacek Banasiak and Wilson Lamb, Phillipe laurencot. Analytic methods for coagulation-fragmentation models. Volume II. Boca Raton, FL: CRC Press
3) Pavel. B. Dubovskiǐ. Mathematical theory of coagulation. Seoul National University.
4) S. Kesavan, Topics in Functional Analysis and Applications, New Age International Publishers, 2015
5) H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011

Course Learning Outcomes: Completion of the course, student will be able to

1. Understand particulate events
2. Can study different aspects of solutions
3. Identify and model efficient and accurate numerical methods
4. Can identify the convergent solutions
5. Different analytical treatment for wider aspect of research

Course Code	MA 867
Title of the Course	FIXED POINT THEORY AND ITS APPLICATIONS
Prerequisite	MA 701, MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Introduce Banach contraction principle and Brouwer – Schauder fixed point theorems.
2. Discuss Ky Fan's best approximation theorem, Prolla's theorem and its extensions.
3. Explore the idea of KKM map principle and its applications.
4. Study fixed point theory in partially ordered spaces and its application in Game theory.

Course Content

Banach Contraction Principle, further extensions of Banach's Principle, The Carisi – Ekeland principle, fixed points of non-expansive and set valued maps, Brouwer- Schauder fixed point theorems.

Ky Fan's best approximation theorems in Hilbert spaces, Applications to Fixed point theorems, Prolla's theorem and extensions.

The KKM- Map principle, Extensions of the KKM-Map Principle and its applications, their variants and application.

Fixed point theorems in partially ordered spaces and other abstract spaces, application of fixed-point theory to Game theory.

Reference Books:

1) M. A. Khamsi, W. A. Kirk, An introduction to metric spaces and fixed-point theory, Wiley-Inter Sci., New York, 2001.
2) W. A. Kirk, B. Sims, Handbook of metric fixed-point theory, Springer, Netherlands, 2001.
3) S. Singh, B. Watson, P. Srivastava, Fixed point theory and best approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, 1997.
4) K. C. Border, Fixed point theorems with applications to economics and game theory, Cambridge University Press, Cambridge, 1985.

Course Learning Outcomes: Completion of the course, student will be able to

1. Understand the concept of non-expansive mapping, contraction principle and Carisi – Ekeland principle.
2. Analyze the application of fixed-point theorem like. Ky Fan's best approximation theorems in Hilbert spaces, Prolla's theorem and. the KKM- Map principle in various fields.
3. Apply fixed point theory in game theory.

Course Code	MA 868
Title of the Course	ADVANCED FUNCTIONAL ANALYSIS
Prerequisite	MA 701, MA 706, MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Discuss some new topologies in Banach spaces and explore compactness of unit ball in these topologies.
2. Introduce the concept of Schauder bases, Pitt's theorem.
3. Explore various properties of classical Banach spaces like c_0, l_p, L_p, C [0, 1].
4. Study the concepts of linear operators, adjoint operators and compact operators in Banach spaces.
5. Learn the spectral theory for operators on a Banach spaces.

Course Content

Topological vector spaces, weak and weak* topologies, The Banach – Alaoglu theorem, Goldstine's theorem, equivalence of reflexivity and weak compactness of the closed unit ball, Schur property of l_1.

Schauder bases, Mazur's basic sequence theorem, block basis sequence, Pitt's theorem, Khintchine's inequality.

Schauder's basis for C [0, 1], Subspaces of l_p and c_0, complemented subspaces of l_p and c_0, Special properties of c_0 , l_1, l_∞ and L_p spaces, basic inequalities.

Linear operators on Banach space, adjoint of a linear operator, compact operators, invariant subspaces, weakly compact operators.

Elementary properties of Spectral theory for operators on Banach space, ideals and quotients, spectrum, The Spectrum of a linear operator, spectral theory of a compact operator.

Reference Books:

1) Walter Rudin, Functional analysis, 2nd edition, international series in pure and applied Mathematics, McGraw-Hill, Inc., New York, 1991.
2) Marian Fabian, Petr Habala, Petr Hajek, Vincente Montesinos Santalucia, Jan Pelant, Vaclav Zizler, Functional analysis and infinite dimensional geometry, CMS Books in Mathematics, Springer-Verlag, New York, 2001.
3) Robert E. Megginson, An introduction to Banach space theory, Graduate Texts in Mathematics, 183, Springer-Verlag, New York, 1998.
4) N. L. Carothers, A Short course on Banach space theory, London mathematical Society student texts, 64, Cambridge university press, Cambridge, 2005.
5) John B. Conway, A course in functional analysis, 2nd edition, Graduate texts in mathematics, 96, Springer-Verlag, New York, 1990.

Course Learning Outcomes: Completion of the course, student will be able to

1. Construct new topologies in Banach spaces where the unit ball is compact with or without any assumption on the space.
2. Finds difference between basis and Schauder's basis and various application of Schauder basis.
3. Understand the various properties of classical Banach spaces.
4. Gain knowledge of various operators in Banach space.
5. Appreciate the Spectral theory for operators on a Banach space.

Course Code	MA 869
Title of the Course	THEORY AND GEOMETRY OF BANACH SPACES
Prerequisite	MA 701, MA 713
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Obtain dualities between embeddings and quotients spaces and introduce fruitful topologies in Banach spaces.
2. Explore the theory of rotundity and its generalization along with some renorming technique.
3. Study the theory of smoothness and its generalization and establish duality relation between rotundity and smoothness.
4. Introduces M-ideals and explore some of its properties.

Course Content

Dual spaces, reflexivity, quotient spaces, locally convex spaces, weak and weak* topologies, The Banach – Alaoglu theorem, Goldstine's theorem, Eberlein – Smulian theorem.

Rotundity, modulus of rotundity, uniform rotundity, Milman – pettis theorem, locally uniform rotundity, weakly uniform rotundity, basic rotund renormings.

Smoothness, duality relation of rotundity and smoothness, modulus of smoothness, uniform smoothness, spherical image map, Frechet smooth, basic smooth renormings.

Introduction to M – Ideals in Banach space and their basic properties.

Reference Books:

1) Robert E. Megginson, An introduction to Banach space theory, Graduate Texts in Mathematics, 183, Springer-Verlag, New York, 1998.
2) Robert Deville, Gilles Godefroy, Vaclav Zizler, Smoothness and renormings in Banach spaces, Pitman monographs and surveys in Pure and applied mathematics, vol.64, Longman scientific and technical, Harlow, 1993.
3) William B. Johnson, Joram Lindenstrauss, Basic concepts in the geometry of Banach spaces, Handbook of the geometry of Banach spaces, vol-1, 1-84, North-Holland, Amsterdam, 2001.
4) P. Harmand, D. Werner, W. Werner, M-ideals in Banach spaces and Banach algebras, Lecture notes in Mathematics, 1547, Springer-Verlag, Berlin, 1993.
5) Joseph Diestel, Geometry of Banach spaces- selected topics, Lecture notes in Mathematics, Vol. 485. Springer-Verlag, Berlin, New York, 1975.

Course Learning Outcomes: Completion of the course, student will be able to

1. Explore the importance of various topologies in Banach spaces and their relations.
2. Analyze the structure of unit ball in a Banach space using the concepts of rotundity and smoothness.
3. Obtain several renorming technique to construct examples and counterexamples for rotund and smooth spaces.
4. Understand the concept of M-ideals in Banach space and its application.

Course Code	MA 870
Title of the Course	FITTED MESH AND FITTED OPERATOR METHODS FOR SINGULAR PERTURBATION PROBLEMS
Prerequisite	MA 707
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. singularly perturbed differential equations
2. fitted mesh methods and fitted operator methods for solving singularly perturbed differential equations

Course Content

Motivation for the study of singular perturbation problems – simple examples of singular perturbation problems – Numerical methods for singular perturbation problems – Simple fitted mesh methods in one dimension

Convergence of fitted mesh finite difference methods for linear reaction – diffusion problems in one dimension.

Convergence of fitted mesh finite difference methods for linear convection – diffusion problems in one dimension

Initial value problems – Some uniformly convergent difference schemes – Constant fitting factors – Optimal error estimates

Boundary value problems – Introduction - Examples – Asymptotic Expansions – Historical Perspective – Necessary conditions– Constant fitting factor

Reference Books:

1) J. J. H. Miller, E. O' Riordan, G. I. Shishkin, Fitted Numerical Methods for Singular Perturbation Problems – Error estimates in the Maximum Norm for Linear Problems in One & Two Dimensions (Revised Edition), World Scientific Publishers, Singapore, 2012.
2) E. P. Doolan, J. J. H. Miller and W. H. A. Schliders, Uniform numerical methods for problems with initial and boundary layers, Boole Press, Dublin, Ireland, 1980.
3) R. E. O'Malley, Introduction to Singular Perturbations, Academic Press, New York, 1974.
4) R. E. O'Malley, Singular Perturbation Methods for Ordinary Differential Equations, Springer – Verlag, NewYork, 1991.
5) P. A. Farrel, A. F. Hegarty, J. J. H. Miller, E. O' Riordan and G. I. Shishkin, Robust Computational Techniques for Boundary Layers, Chaman & Hall/CRC, 2000.

Course Learning Outcomes: Completion of the course, student will be able to

1. construct fitted mesh and fitted operator methods for various classes of singular perturbation problems
2. estimate local and global bounds on the error of the constructed numerical method

Course Code	MA 871
Title of the Course	MATHEMATICAL THEORY OF WATER WAVES
Prerequisite	MA 707
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Learn the basic characteristics of water waves
2. Learn the mathematical formulation
3. Study the approximate theories
4. Analysis wave phenomenon in various conditions
5. Learn a few mathematical methods

Course Content

Basic Hydrodynamics**:** The laws of conservation of momentum and mass, Helmholtz's theorem, Potential flow and Bernoulli's law, Boundary conditions, Singularities of the velocity potential, Notions concerning energy and energy flux, Formulation of a surface wave problem.

The Two Basic Approximate Theories: Theory of waves of small amplitude, Shallow water theory to lowest order. Tidal theory, Systematic derivation of the shallow water theory.

Waves Simple Harmonic in the Time: Standing waves, Simple harmonic progressing waves, Energy transmission for simple harmonic waves of small amplitude, Group velocity, Dispersion relation, and Simple harmonic oscillations in water of constant depth.

Waves Maintained by Simple Harmonic Surface Pressure in Water of Uniform Depth: Forced Oscillations, the variable surface pressure is confined to a segment of the surface, Periodic progressing waves against a vertical cliff.

Waves on Sloping Beaches and Past Obstacles: Two-dimensional waves over beaches sloping at an angle, Three-dimensional waves against a vertical cliff, Waves on sloping beaches, Diffraction of waves around a vertical wedge. Sommerfeld's diffraction problem, Brief discussions of additional applications and of other methods of solution.

Reference Books:

1) Stoker, J. J., Water waves, New York: Interscience Publishers, 1957
2) Dean, R. G. and Dalrymple, R.A., Water wave mechanics for Engineers and Scientists, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1994
3) Whitham, G. B., Linear and Nonlinear Waves, John Wiley & Sons, 2011
4) Chiang C. Mei., The applied dynamics of ocean surface waves, Wiley-Interscience, 1983
5) Lamp, H., Hydrodynamics, Cambridge University Press, 1993

Course Learning Outcomes: Completion of the course, student will be able to

1. Formulate various wave phenomenon in water
2. Find wave characteristics
3. Apply mathematical techniques to solve problems
4. Analyze wave phenomenon
5. Develop mathematical method for a given problem

Course Code	MA 872
Title of the Course	INTRODUCTION TO HYDRODYNAMIC STABILITY
Prerequisite	MA 703, MA 704
Credits (L-T-P)	3 (3 – 0 – 0)

Course Learning Objectives: Objective of the course is to

1. Introduce the concept of stability of fluid flows
2. Introduce the basics of linear stability theory
3. Examine several instabilities such as, Rayleigh-Taylor, Kelvin-Helmholtz instability etc.
4. Discuss and derive Rayleigh and Orr-Sommerfeld equations
5. Discuss numerical techniques for computationally determining stability

Course Content

Basic review of fluid dynamics: Navier-Stokes equations for Newtonian, non-Newtonian fluids.

Bifurcations, phase space, and phase portrait; stability and instability, linearized problems and normal mode analysis. Kelvin-Helmholtz and Rayleigh-Taylor instabilities, Rayleigh-Benard convection (thermal instability), Taylor-Couette centrifugal instability.

Definition of stability, disturbance equation, critical Reynolds numbers, development of instabilities in space and time etc.

Stability of parallel shear flows: inviscid theory and viscous theory, Spatio-temporal stability analysis: absolute and convective instabilities. Computational methods for hydrodynamic stability.

Reference Books:

1) P. Drazin, Introduction to hydrodynamic stability, Cambridge.
2) S. Chandrasekhar, Hydrodynamic and hydromagnetic stability
3) Francois Charru, Hydrodynamic Instabilities, Cambridge.
4) Stability and Transition in Shear Flows" by Schmidt and Henningson, Springer
5) Theory and Computation in Hydrodynamic Stability" by Criminale, Jackson and Joslin

Course Learning Outcomes: Completion of the course, student will be able to

1. Develop an understanding by which "small" perturbations grow, saturate and
2. modify fluid flows.
3. Apply linear stability to determine the onset of instability.
4. Determine the nature of instabilities.
5. Recognize importance of unstable nonlinear solutions.
6. Build a foundation for addressing/approaching research problems on laminar to turbulent transition.